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Don Harold Morris
Iowa State University

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Photomechanics techniques for elasto-plastic stress analysis

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Don Harold Morris

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I. INTRODUCTION

Most transparent materials become doubly refracting when subjected to load. This fundamental property of double refraction is the basis of the photomechanics technique, where a transparent model is loaded and examined in polarized light. The result is an optical interference pattern of light and dark bands, called fringes. The optical effect may be due to stress, strain, or a combination of the two, depending on the material.

The branch of experimental stress analysis that makes use of double refraction in certain transparent materials is called Photomechanics. Photomechanics may be subdivided into several branches - photoelasticity, photoviscoelasticity, dynamic photoelasticity, photothermoelasticity, photoplasticity, etc. Photoelasticity is the most advanced of these branches. The method of photoelasticity is an accurate experimental tool for two and three-dimensional stress analysis studies.

The optical effects, or fringes, observed in a field of polarized light are called isochromatics and isoclinics. In photoelasticity the isochromatics are proportional to the difference between the principal stresses and the isoclinics give the directions of the principal stresses. An understanding of the quantitative relationship between the stress state and the resulting optical effects provides a useful and precise experimental means for studies of relatively complicated stress states.

Experimental evidence indicates that the optical effects produced by stress and/or strain in transparent materials exist even in the case of inelastic deformations. Thus it appears that the problem of evaluating elastoplastic states of stress and strain is possible. In order to realize this possibility it is necessary to establish a quantitative relationship between the optical effects and the state of stress and/or strain which is operative beyond the elastic limit of the material used.

A review of the literature indicates that some papers report theoretical and/or quantitative experimental results, while others contain only qualitative observations. The methods and interpretations of data do not indicate a unified treatment such as those of photoelasticity. Some papers give only partial information about the optical effects, that is, a quantitative meaning is given to the isochromatics while no reference is made to isoclinic data. In addition, some authors give data only for uniaxial tensile tests and do not discuss biaxial states of stress. There is a wide range of testing methods and interpretations of data used by the various researchers. Some of the differences may be attributed to the fact that birefringence (for inelastic deformations) is caused by stress for some materials, while for other materials birefringence is due to strain.

The object of this investigation was to develop a technique or method for elasto-plastic stress analysis using the optical effects of transparent materials. The technique should be developed to the point where it can be used to give valid experimental predictions of plastic stresses or strains. Of paramount importance was the selection and characterization of a suitable model material. Birefringence must exist in the elastic and plastic states of stress, and the relationship between stress and/or strain should be single valued. Moreover, the material must exhibit the appropriate mechanical behavior from the point of view of similitude requirements for transition from model to prototype.

The investigation was limited to two-dimensional states of stress. Several materials were studied; the suitable material selected should be readily obtainable by other investigators. In addition, the material should be castable in large blocks in order that three-dimensional studies may be made.

II. REVIEW OF LITERATURE

Although birefringence in the inelastic range had been observed in the early 1900's by Filon (1), Coker and Chakko (2), and Filon and Jessop (3), it was not until the early 1950's that much additional published work appeared.

Hetényi (4) used the results of tensile tests to study a nylon copolymer. He found that the isochromatics represented the strain distribution, but gave no mention of isoclinics. Emphasis was placed on the optical indication of yielding, and it was found that unidirectional slip bands formed at low humidity (14%), while a symmetrical yield pattern formed at high humidity (46%).

Fried (5) made some observations on photoelastic materials stressed beyond the elastic limit. The materials studied were polystyrene, lucite, plexiglass, nylon, cellulose acetate, silver chloride, and cellulose nitrate (celluloid). Fried found only cellulose nitrate suitable for his proposed method of study. He did not try to determine whether the optical effects were caused by strain or stress, but was only interested in their functional relationships.

Bayoumi and Frankl (6) derived a mathematical relationship between relative retardation and the stress and strain differences. Equations are given for two cases: coincidence of principal directions of stress and principal directions of strain, and principal directions of stress and strain not coincident. Bayoumi and Frankl used their data, and data given by others, and concluded that a given state of birefringence is completely defined by the stress and strain, and does not depend upon the temperature of the test nor upon the path of loading. All results are based on data from tensile and creep tests, using Catalin 800, C.R.-39, Marbelette, and different types of Bakelite resins.

Fried and Shoup (7) used polyethylene to study the photoelastic effect in the region of large deformation. They found that the optical retardation varied linearly with the principal strain difference well beyond the linear range between stress and strain. The shear strain distribution in a photoplastic model was compared with that found in an aluminum prototype (tensile specimen with a central circular hole), resulting in good agreement between model and prototype.

One and two-dimensional studies of celluloid were conducted by Itō (8). The results indicated that, for both one and two-dimensional states of stress, yielding is caused by maximum shearing stress.

Studies on plastic deformation, using celluloid, were conducted by Nisida, Hondo, and Hasunuma (9). Tension-compression calibration tests gave the relationships between the optical retardation, stress, and strain. The results of the calibration tests were applied to three experiments: (a) elastic-plastic bending of a beam, (b) indentation by a wedge, and (c) compression of a wedge by a flat die. Good agreement was obtained between experimental results and the theory of plasticity.

The most extensive study of photoplasticity has been that of Frocht and co-workers (10-16). They used cellulose nitrate (celluloid) as the model material. Experiments indicate that, for the same conditions of temperature, relative humidity, time, and optical path, birefringence is a function of the principal stresses and their histories. It was also found that the isoclinic parameters represent the directions of the secondary principal stresses regardless of the state of stress, and the histories of the stresses. These results were used to determine, by photoplasticity methods, factors of stress concentration for plates with holes, notches and fillets. Good agreement was obtained between the photoplasticity methods, and theoretical and other experimental methods.

Celluloid was also used by Mönch and Loreck (17) to study the accuracy and limits of application of plane photoplastic experiments. Use is made of the anomalous dispersion of birefringence that occurs in celluloid, that is, the relative retardation depends on the wavelength of light used. The authors used red light (6550 Å) and blue light (4360 Å). It was found that the product of the fringe order and wavelength is essentially constant for elastic stresses, but it differs when plastic flow begins. This dispersion can be used to evaluate stress and strains at free boundaries, and to determine the boundary between the elastic and plastic states of stress.

Hetényi (18) gives a review of the developments in photoplasticity during the 1950's. An extensive bibliography is also given. Zandman (19) reviews the literature on the same subject in the Soviet Union and Democratic Republics.

Itō (20) and Rao (21) discuss the possibilities of using certain materials for studies in photoplasticity. Itō indicates that polycarbonate should be suitable, while Rao reviews the work of other researchers.

Gurtman, Jenkins, and Tung (22) studied a polycarbonate polymer for possible use in photoelastoplastic studies. Tests were performed in both uniaxial and biaxial stress fields. Birefringence was found to be linear with principal strain difference for vanishingly small strain rates, and the isoclinics were found to align themselves with the principal stress directions. Tests conducted at constant strain rate enabled the authors to develop an analytical expression that relates stress and strain. Another relation correlates current values of stress and strain in the material with the stress and strain rates.

Polycarbonate was the material selected by Brill (23) for basic studies in one and two-dimensional photoplasticity. He concluded that the birefringence is a function only of the principal strain difference. The author also found that the isoclinic parameter was in good agreement with the principal strain directions. A tensile specimen with a central circular hole was used to check the photoplastic results with tests on metallic prototypes, with good agreement resulting between the model and prototype.

Hunter and Schwarz (24) and Hunter, Wilshaw, and Kyser (25), made photoelasto-plastic studies that differ in method from any of those previously mentioned. Their method utilized the creep and frozen stress characteristics of epoxy resins. A non-linear stress strain curve is obtained when uniaxial tensile specimens, under constant stress, are subjected to a thermal cycle with a maximum temperature significantly less than the critical temperature of the model material. These same tensile specimens are also used to obtain a birefringence-stress curve. The results of the calibration tests were used to study elasto-plastic stress distributions in infinite plates with centrally located holes, stress distributions in the vicinity of a simulated crack, and stress distributions in notched bars subjected to bending.

Whitfield (26) made studies to characterize polycarbonate as a photoelasto-plastic material. The characterization tests were performed in the form of uniaxial strain rate, creep, isotropy, isoclinic, and reloading tests. The yield locus was determined, and the results indicated that although polycarbonate is not a Mises material, the yield locus may be approximated by a circle over a substantial range of Lode's variables. Tests conducted on large plates with holes gave excellent correlation with two-dimensional theory in the elastic zone and in the plastic zone where thickness change is small. Substantial divergence between theory and experiment occurred in the plastic zone where thickness changes were large.

III. MATERIAL DEVELOPMENT

A suitable material must have a stress-strain relationship similar to that of a real engineering material. The mechanical behavior of various materials must be studied to determine if such a material exists. Application of the material to elastoplastic problems using photomechanics techniques implies that optical characterization tests must be performed.

The materials studied, a description of the experimental apparatus, and the material casting procedure are presented in the following sections.

A. Experimental Apparatus

Material characterization tests were carried out using a tensile machine capable of controlled strain rates. The machine was a converted, hand-operated tensile tester powered by a 1/12-hp electric motor with a variable speed output. The machine's cross-head and power supply were coupled by a speed reducer (50:1 was found to give appropriate strain rates).

Instrumentation was built that provided for the simultaneous recording of load, strain, and birefringence. The design of the recording apparatus is described below.

Load measurements were made through use of a 1000-lb load cell and indicator. Values of load at any desired instant of time were hand recorded on a two-channel Brush pen type recording oscillograph. It was felt that better accuracy could be obtained in this manner than if the indicator signal was used to operate one channel of the oscillograph. All loading rates used in the investigation were low enough such that the above described method of load recording was possible.

An amplified signal that caused blips on one channel of the oscillograph was used to record strain as a function of time. The signal was due to a microswitch that was activated at each turn of a handle on the tensile machine. The relationship between strain and blips was determined by calibration. The calibration procedure consisted of measuring the distance between finely scribed transverse lines on a tensile specimen as a function of number of handle turns. A traveling microscope with 0.0001 inch scale divisions was used for distance measurements. By employing the definition of strain it was possible to relate handle turns (blips) to strain. Several tests were run, and the average was used to give a linear relationship between strain and blips.

The tensile specimens used for material characterization are under a uniform load, and thus have uniform birefringence. With increasing strain the specimen will appear alternately dark and light when viewed in the field of a polariscope. Since load and strain are measured as a function of time, it would be convenient to measure birefringence as a function of the same parameter. This was accomplished by using a photocell to pick up the cyclic variation in light intensity, amplifying the signal, and feeding the resulting signal into the second channel of the oscillograph. The photocell was built into a small polariscope that straddled the test specimen. The fringe orders were thus determined to one-half order, as a function of time.

Since birefringence, strain, and load were recorded simultaneously as a function of time, it was possible to eliminate time as a factor in the stress-strain, stress-birefringence, and strain-birefringence calibrations.

B. Materials Investigated

A prerequisite to the investigation of any material is a knowledge of the general material requirements. These requirements are presented in this section, along with results of tests on epoxies and polyesters.

1. General material requirements

Many factors influence the use of a birefringent material for elasto-plastic studies. Some of the factors are the same as those of materials used for photoelastic studies. A list of ideal properties for photoelastic materials may be found in a book by Dally and Riley (27), and will not be repeated here. There are several other factors, including the list mentioned above, that affect the use of a material for elasto-plastic studies.

Frocht and Thomson (14) give five requirements that are necessary for transition from model to prototype in plane elasto-plastic problems. In addition to similarity of geometry and loading the requirements are:

(a) the materials of model and prototype must have the same shape of stress-strain curve, and if some points in the model undergo unloading during the application of the external loading then the stress-strain curve must be the same for both loading and unloading;

(b) the law of yielding must be the same for both materials;

(c) the value of Poisson's ratio in the plastic range must be the same for both materials.

The five conditions listed include three that involve material characteristics. Whether or not these requirements are met will be discussed in a later section.

In addition to the above requirements, the material must be birefringent in both its elastic and plastic states. For ease of application the relationship between birefringence and stress and/or strain should be single valued. Moreover, the material must exhibit sufficient plastic flow before fracture, and should be useable in a tension-compression field.

The above requirements pose quite a problem. The difficulties become apparent through a search of the literature. Some materials are available only in thin sheets, and some have complicated relationships between their optical and mechanical behavior. Nonetheless, all of the materials display varying degrees of merit.

With all requirements in mind, the present investigation was undertaken to try to overcome some of the problems associated with previous studies.

One of the main problems of material suitability, i.e., material properties, may be alleviated by considering materials whose properties are easily altered. Two classes of materials, namely the epoxies and polyesters, exhibit property variability. Varying the amounts of resin constituents and hardeners results in castings with almost limitless material properties. In addition, specimens of any desired size may be cast.

2. Epoxies

A large number of different epoxy resins are manufactured in this country. The ones considered in this investigation represent only a very small number of those that are available. Test results are not necessarily indicative of general epoxy behavior. In fact, other epoxies may exist which give results that are far superior to those investigated.

a. Preliminary tests Preliminary testing consisted of subjecting 1/4-in. thick tensile specimens (Fig. 1) to a constant strain rate. During the course of a test several quantities could be observed qualitatively to determine whether or not further testing was warranted. The load indicator provided the general shape of the stress-strain curve, in particular it gave an indication of the size of the plastic region and the amount of strain hardening. Viewing the specimen in a 15-in. polariscope showed the optical response of the material. The photocell-polariscope arrangement previously mentioned was not used in preliminary testing.

The first preliminary test was conducted on an epoxy material proposed by Sampson (28). The applicability of this material for elasto-plastic studies had not been previously considered. Sampson's formulation consisted of 50 pbw (parts by weight) Bakelite ERL 2774 resin, 50 pbw Bakelite ERL 2795 resin, and 25 pbw Bakelite ZZL 0803 hardener; cured to Sampson's specifications. The casting technique is given in a later section. This material exhibited very little nonlinear deformation and was dropped from further consideration, even though different resin-hardener ratios might give more favorable mechanical response.

Ciba 502 resin and 10 pph (parts by weight of curing agent per hundred parts by weight of resin) Ciba HN951

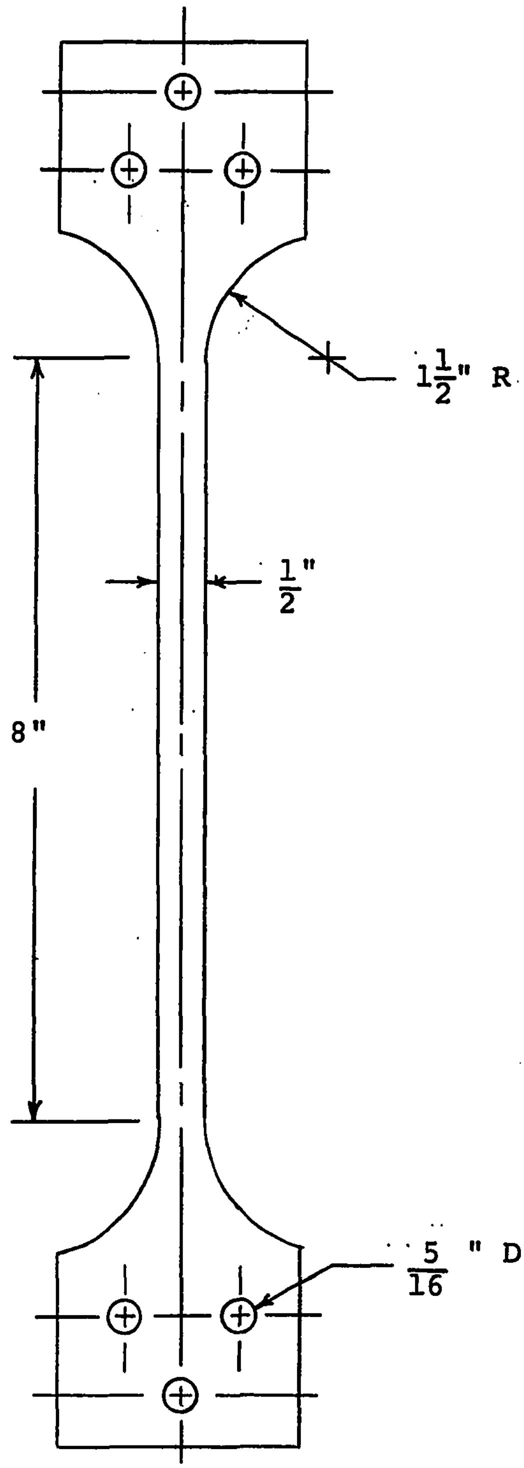

Fig. 1. Uniaxial test specimen

hardener was the next epoxy studied. The material was cured at 50°C for 48 hours. Qualitative tests indicated only a small non-linear mechanical response before fracture.

Very large inelastic deformations were observed when 80 pbw Ciba 502 and 20 pbw Bakelite ZZL 0803 (cured at 80°C for 24 hrs.) was tested. The load reached a peak value, then continually decreased. Upon reaching the peak load the definition of the fringe orders deteriorated so that one fringe order could not be distinguished from another. This material was not considered for further testing.

Another epoxy formulation consisted of 70 pbw Ciba 6005 resin and 30 pbw Ciba Lancast A hardener. As in the previous material, the load reached a peak value with the same result in optical response. Further studies were not made.

The last epoxy studied was Ciba 502 resin cured with 35 pph Lancast A. The mechanical-optical response, in general, was the same as for the two previous materials.

Of the five epoxies considered in preliminary testing, the last three exhibited the same general mechanical-optical behavior. Two of them used the same resin, but different hardeners. Optically, both materials are of no value beyond the maximum load, since fringe orders can not be accurately identified. Two used the same hardener, with the same result in optical response. The approximate strain at peak load was maximum for the 502-Lancast A mixture. This strain could be calculated from the known strain rate and approximate time to peak load. Thus, this material was considered for quantitative testing to gain some insight into its applicability as a material for the solution of problems with limited inelastic deformations. The results are presented in the next section.

b. Characterization tests A simple test to perform, yet one that gives an indication of the usefulness of a material for elasto-plastic studies, is a uniaxial tension test. All characterization tests were performed in this manner. Three sets of data were taken: fringe order, load, and strain. The method of recording data was explained in the section Experimental Apparatus.

Characterization tests were conducted only on the Ciba 502-Lancast A mixture. The curing cycle consisted of room temperature cure for 24 hrs. and post cure at 80°C for 16 hrs. The casting was removed from the mold and hung in an oven for the post cure.

Stress-strain, birefringence-stress, and birefringence-strain curves are shown in Figs. 2, 3, and 4, respectively. No attempt was made to determine Poisson's ratio.

The stress-strain curves reveal that an upper bound on the strain rate exists, below which the curves are strain rate independent, at least for the reasonable strain rates considered. The importance of this independence will be

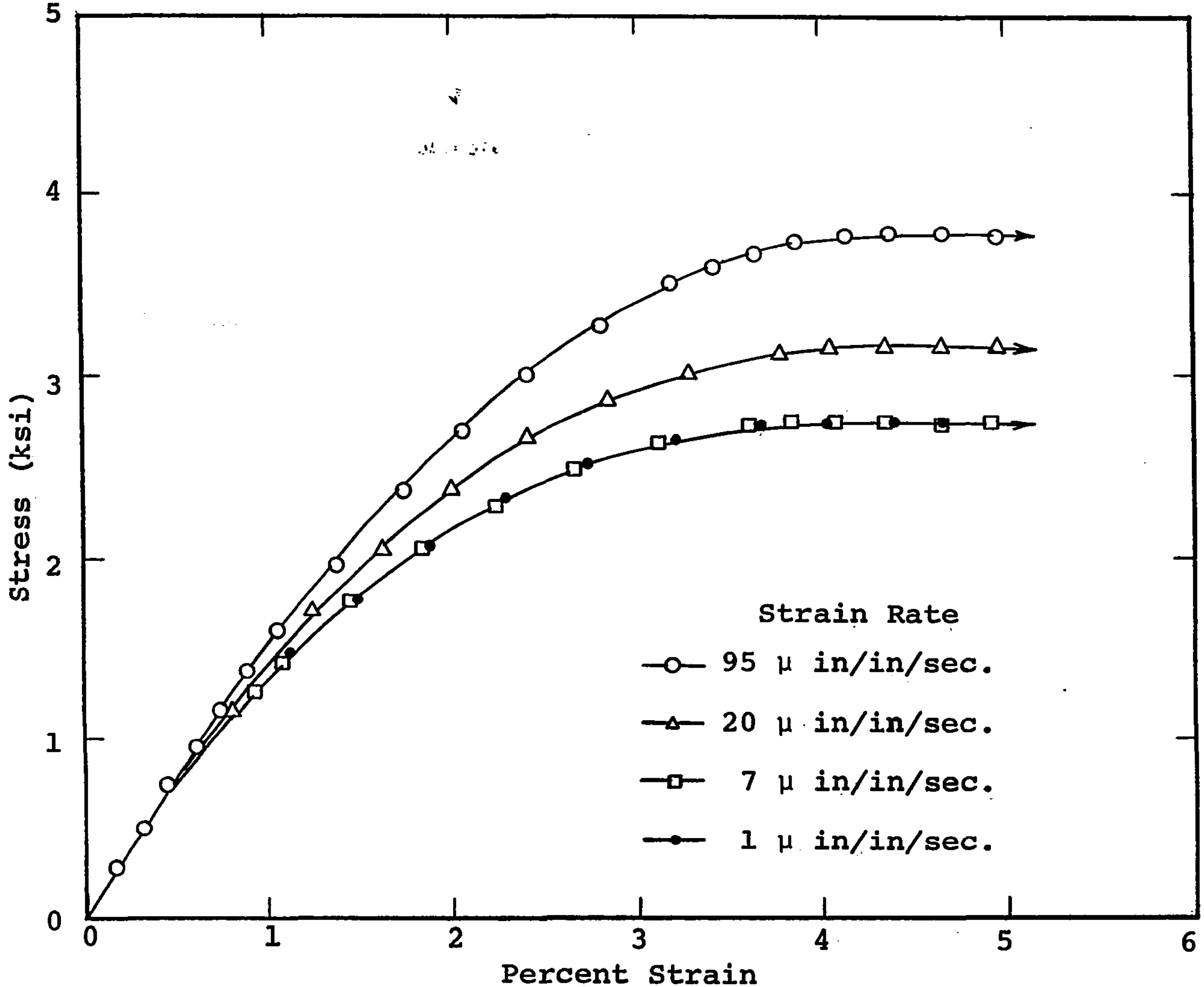

Fig. 2. Stress-strain curves for Ciba 502-Lancast A epoxy

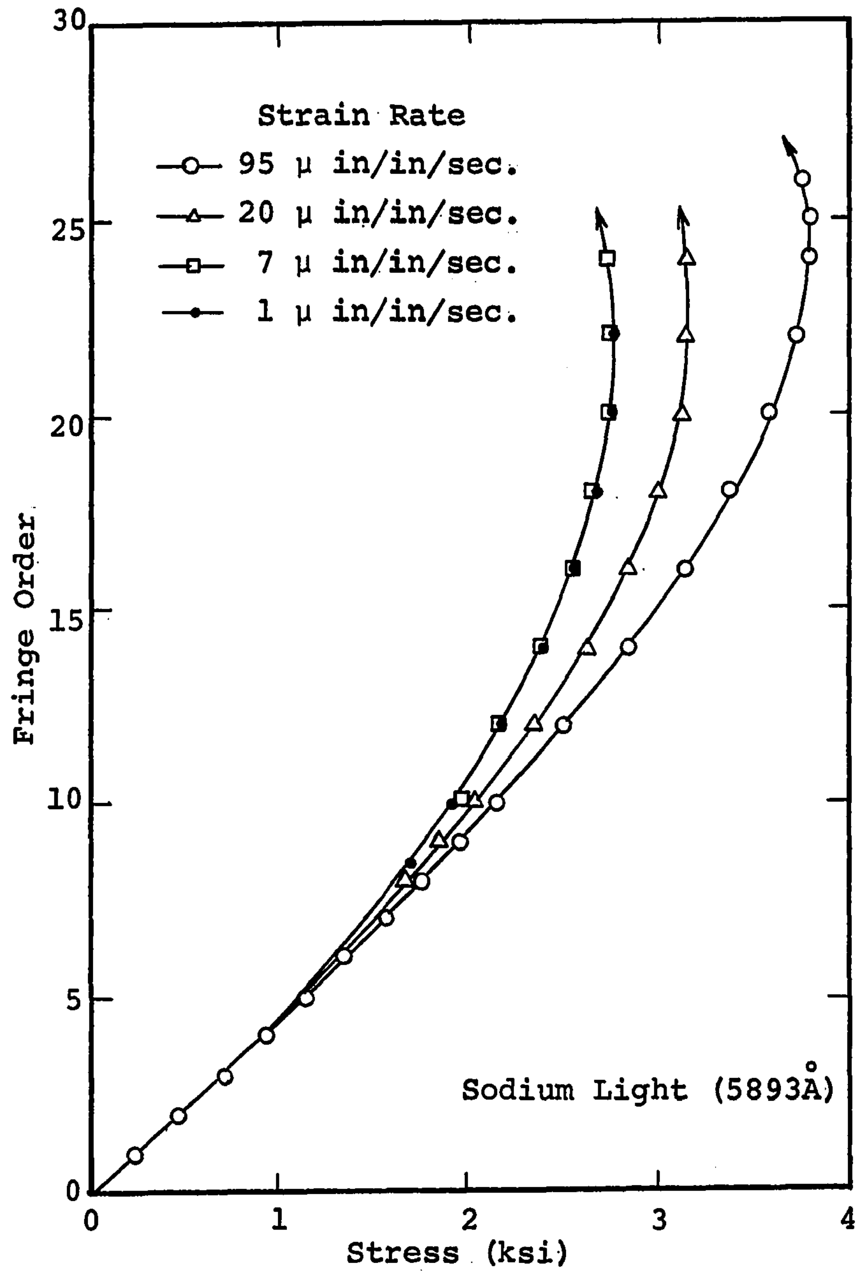

Fig. 3. Birefringence-stress for Ciba 502-Lancast A epoxy

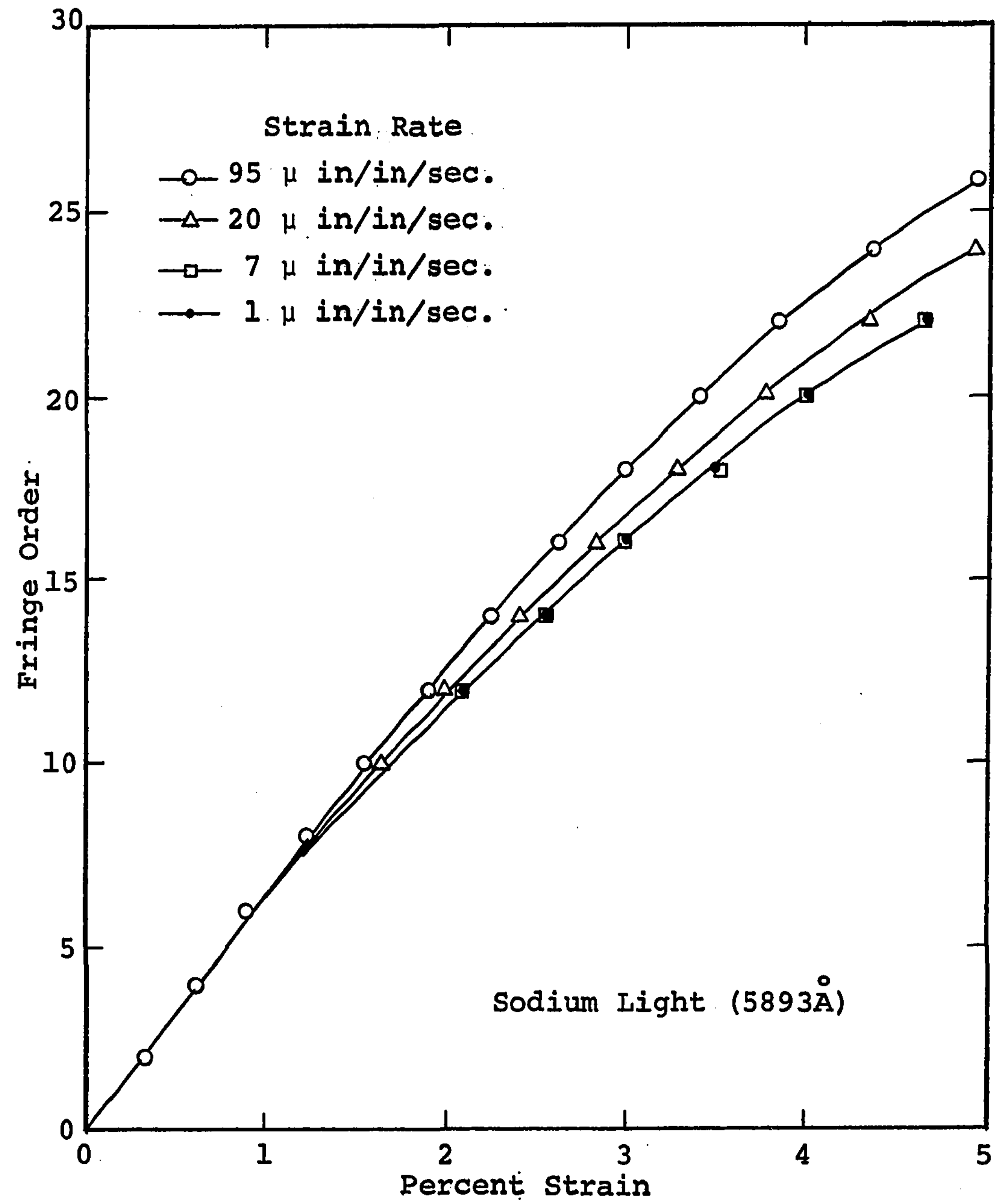

Fig. 4. Birefringence-strain for Ciba 502-Lancast A epoxy

subsequently discussed in the section concerning polyesters.

Inspection of the fringe order-stress and fringe order-strain curves also show a non-linear relationship between birefringence and stress or strain. For all curves stress and strain are nominal values.

Further studies into the applicability of this material were not made, as the primary purpose of the investigation was to find a material that would permit problems of very large strains (at least 10%) to be solved. However, this epoxy material might be useful for problems with strains not exceeding about 4%. Note that the strains referred to are model strains.

As previously mentioned, one material requirement is that the stress-strain curve of model and prototype have the same shape. A comparison between the shapes of the stress-strain curves can be made by means of the Ramberg-Osgood equation (29),

$$\frac{E\varepsilon}{\sigma_1} = \frac{\sigma}{\sigma_1} + \frac{3}{7}\left(\frac{\sigma}{\sigma_1}\right)^n \qquad (1)$$

where E is Young's modulus, σ_1 is a secant yield stress defined by a secant modulus $E_s = 0.7E$, σ is stress, ε is strain, and n is a parameter chosen to provide the best fit to the stress-strain curve of the actual material under consideration. The Ramberg-Osgood equation simplifies the stress-strain curves so that the difference between a wide variety of materials is expressed solely by the exponent n, which is a function of the shape of the diagram. According to Frocht and Thomson (14), some types of steel and aluminum have stress-strain curves which fall in the range n = 9 to n = 19. Figure 5 shows a comparison between the stress-strain curves for n = 9 and n = 19 and the curve for Ciba 502-Lancast A.

Thus, it appears that further testing of this epoxy material is warranted. Also, further characterization tests on the 502-ZZL and 6005-Lancast A materials might yield meaningful Ramberg-Osgood stress-strain curves.

3. Polyesters

Two different brands of polyesters marketed in the United States were investigated. They are: "Paraplex" by Rohm and Haas and "Laminac" by American Cyanamid. Both products are available in flexible and rigid resins. Blending the two grades of resins enables one to obtain properties that are intermediate to those of the individual types. In addition, the resins can be cast into blocks of any desired size allowing the possibility of three-dimensional studies.

a. Preliminary tests Rohm and Hass manufactures several types of polyester resins, of which one rigid and one flexible resin was selected for preliminary testing. The flexible resin was Paraplex P-13 and the rigid resin was Paraplex P-47. Individual cure of the

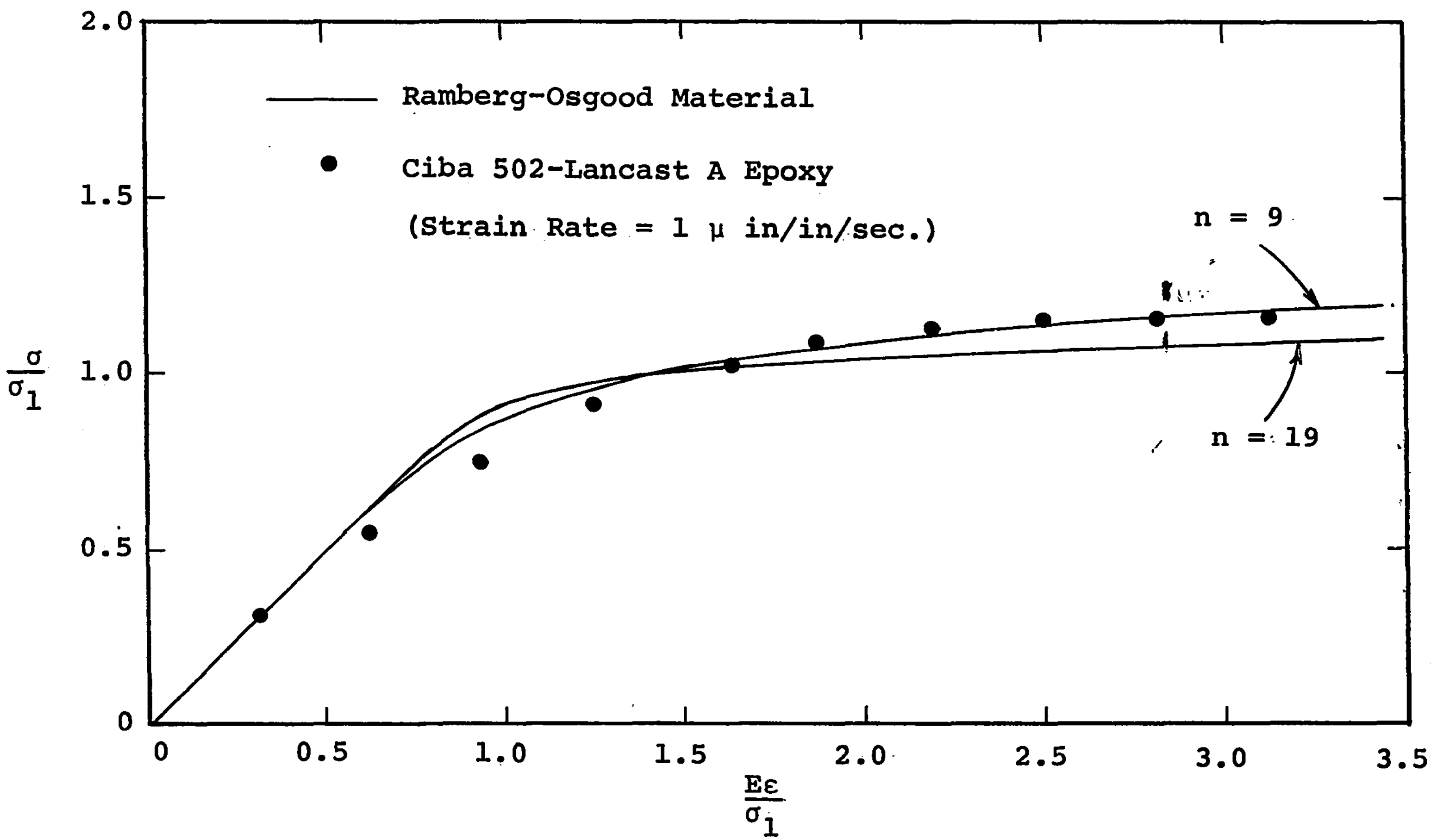

Fig. 5. Non-dimensional stress-strain curves for Ciba 502-Lancast A epoxy and Ramberg-Osgood materials (n = 9, 19)

materials resulted in a clear rigid casting, but a flexible casting that exhibited a milky haze. Combination castings also exhibited the haze, but to a lesser extent the higher the percentage of rigid resin. Light transmission was reduced due to the haziness resulting in a distinct possibility that three-dimensional studies might be hampered. The Laminac series did not exhibit the milky haze, thus attention was focused on this material since a prime objective of the investigation was to find a material capable of use for three-dimensional work.

The Paraplex resins might be satisfactory for two-dimensional studies. Therefore, a few comments will be made concerning some aspects of specimen casting and stress-strain and birefringence-strain behavior.

The Paraplex series of resins were studied prior to the Laminac series, and some initial difficulties were encountered in casting Paraplex specimens. Benzoyl peroxide powder served as the catalyst, and required an elevated temperature before gellation occurred. Castings frequently stuck tenaciously to the mold rendering mold removal difficult. Many castings were not useable. Silicones and ordinary paste wax were used as a mold release agent throughout the Paraplex investigation. The reader is referred to the section entitled Casting Technique for the method found to give good quality castings.

No claim is made that the above mentioned release agents will not give good castings; more experimentation may prove useful. But, initial work with Laminac resins yielded an easy, reliable method of producing good quality castings and no further work was done concerning the previously discussed method.

The stress-strain curves shown in Fig. 6 are for flexible and rigid materials and also for a 40:60 mixture of flexible and rigid resins. The 40:60 mixture reveals strain hardening to fracture at a strain of about 7%. The non-linear range is greater than that of the 502-Lancast A epoxy, but the optical response is much less satisfactory. Birefringence-strain is depicted in Fig. 7. Two limitations are apparent: low optical response and a multi-valued birefringence-strain relationship. This multi-valued characteristic will be discussed in detail in a subsequent section.

The low optical response might prove desirable in some cases, for example, in isoclinic determinations. This aspect of the problem is left for other investigations, as is the study of other flexible-rigid resin combinations.

Individual rigid and flexible castings, as well as combination castings, of the Laminac series of polyesters did not exhibit the milky haze of the Paraplex series. Three-dimensional work is possible, thus emphasis was switched to the Laminac materials. Preliminary tests indicated that various combinations of rigid and flexible resins gave a wide

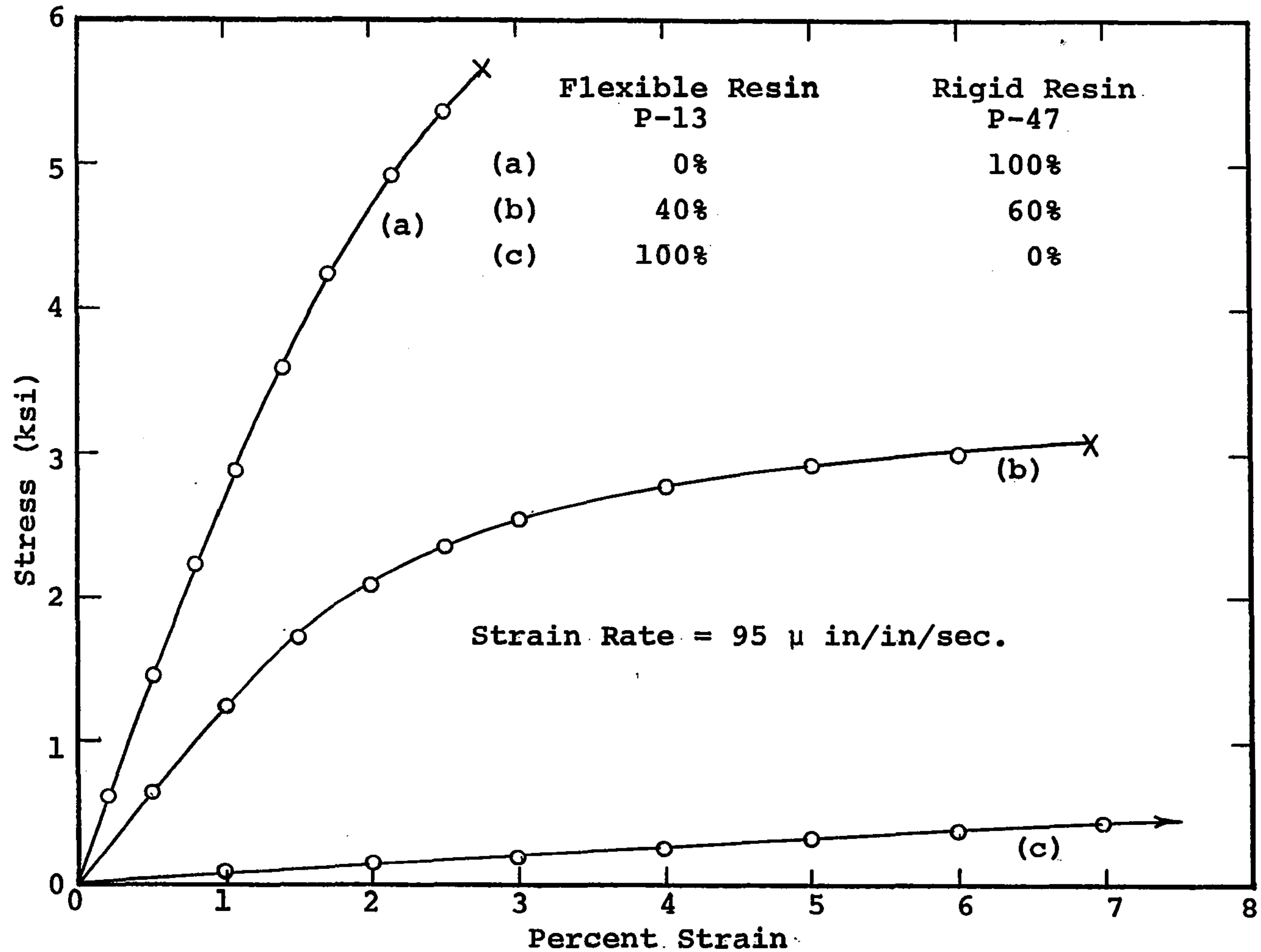

Fig. 6. Stress-strain curves for Paraplex polyester resins

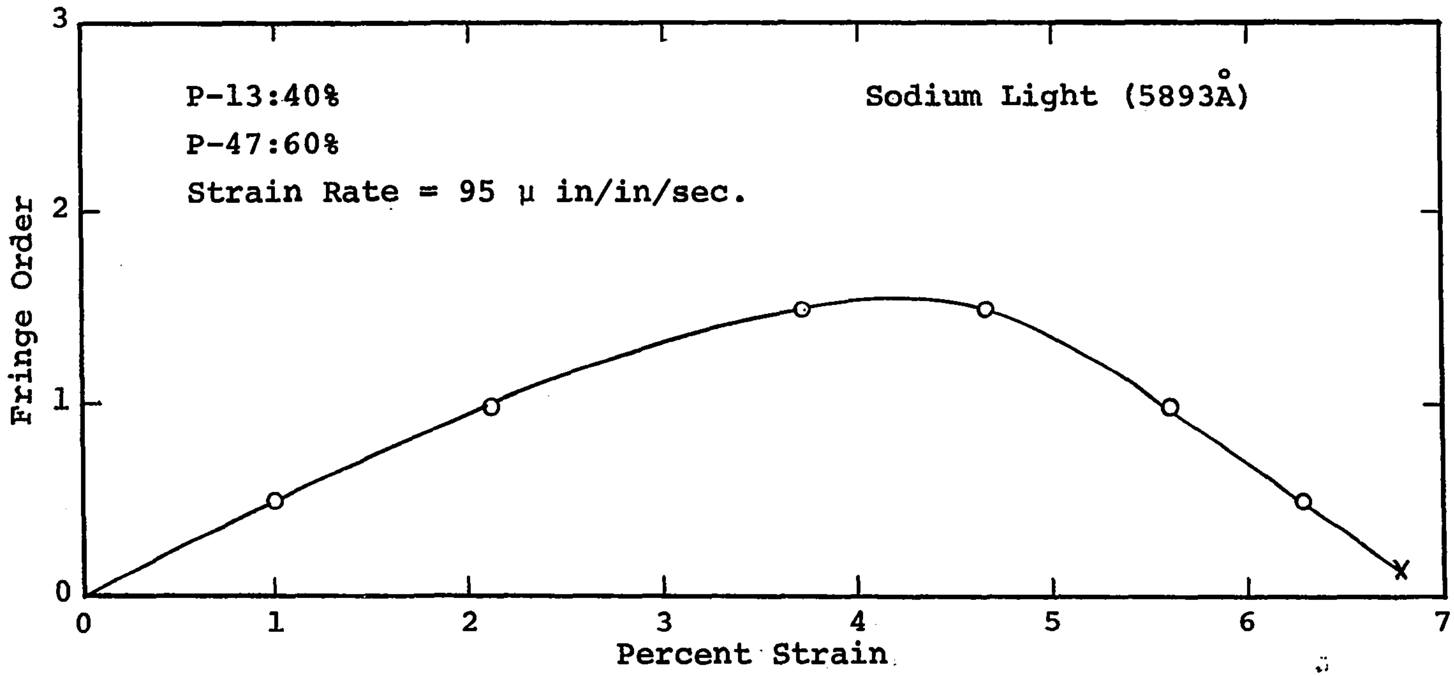

Fig. 7. Optical response for 40:60 mixture of flexible and rigid Paraplex polyester resins

range of material properties, and large strains before fracture. Test results are presented in the following section.

b. <u>Characterization tests</u> This section is devoted to a discussion of the Laminac series of resins. Of all materials investigated, this polyester appears to be best suited for the solution of elasto-plastic problems in the region of large plastic strains (at least 10%).

A few preliminary remarks will be made concerning the method of casting as the final method was not so easily perfected as indicated in the section Casting Technique. Both the Paraplex and Laminac resin series were initially cured with benzoyl peroxide. Gellation occurred only at elevated temperatures. After several unsuccessful attempts at casting Laminac resins it was observed (through a class door in the curing oven) that at about gell temperature castings stuck to some parts of the glass mold, but released at other parts. This occurred with silicone grease or wax as release agents, resulting in castings with rough surfaces and residual stresses. The same result was observed when a different mold release agent, ReleasaGen H-15-1, was used. The problem was solved by using a catalyst that caused gellation at room temperature, methyl ethyl ketone peroxide (MEKP). Blending 1% MEKP with the resins gave a mixture with a pot life of about one hour. The resultant mixture was cured at room temperature for 24 hrs, then removed from the mold. Post cure was obtained by hanging castings in an oven, raising the temperature at a rate of 4°C per hour, holding a constant temperature of 80°C for 16 hrs, and lowering the temperature at a rate of 4°C per hour. Different thermal cycles might give stress-strain curves other than those obtained, but this aspect of the problem was investigated in only a limited fashion.

Comparison of stress-strain curves for materials cured in the manner discussed above, and materials cured at room temperature for one week revealed that high temperature cure gave an increase in stress for the same strain. No further thermal cycle studies were made. All remaining tests were conducted on materials cured at elevated temperature, as described above.

The flexible resin used was Laminac EPX-126-3 and the rigid resin was Laminac 4116. A stress-strain curve for each of these materials is shown in Fig. 8. The stress-strain response of the rigid component is nearly linear to fracture at a stress of 8200 psi. The flexible component shows a non-linear response, while fracture did not occur for the strains shown.

The birefringence-strain behavior for combination mixtures is similar to that of the Paraplex series. Birefringence is initially positive, reaches a peak, then decreases

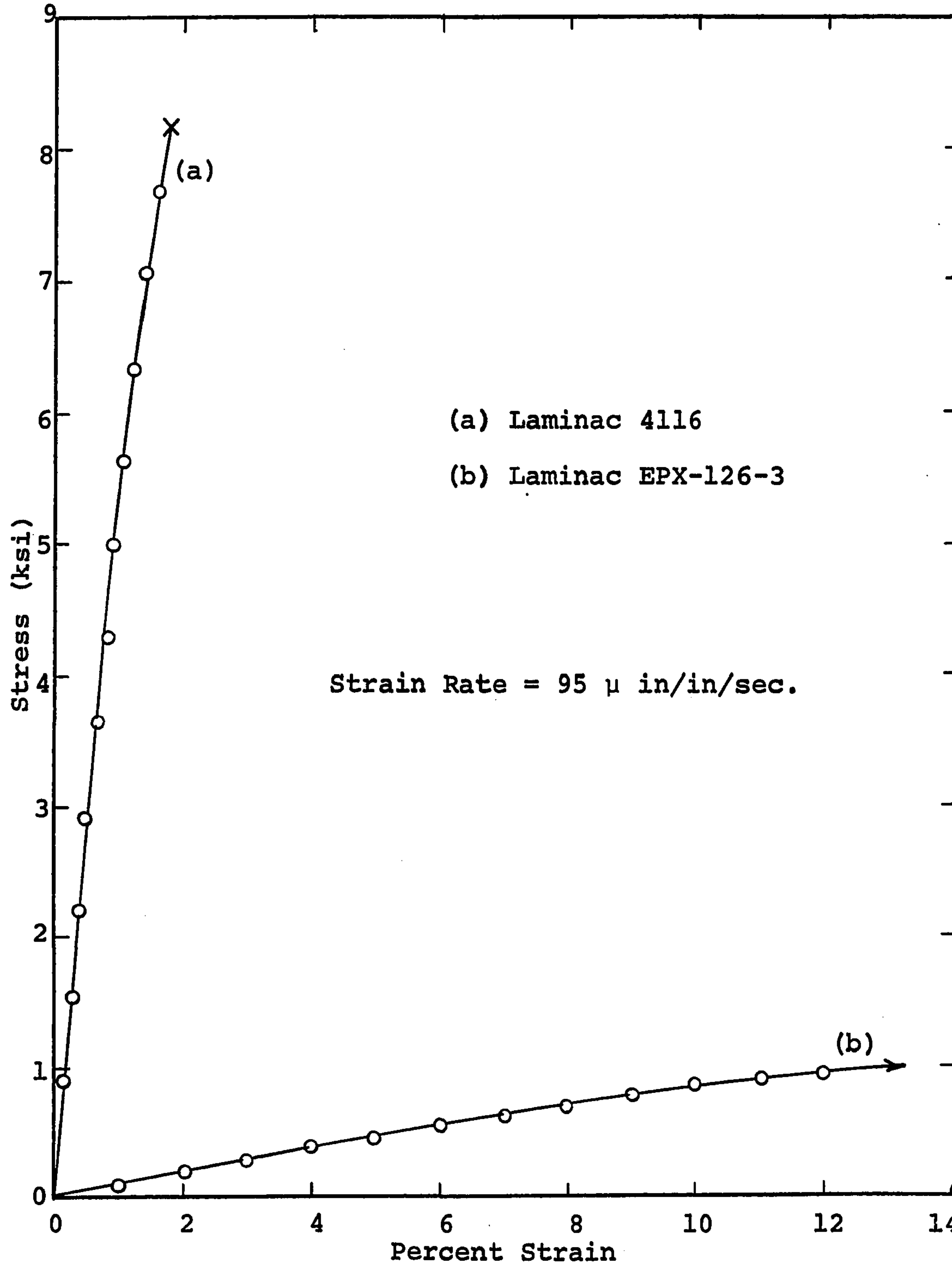

Fig. 8. Stress-strain curves for Laminac polyester resins

and for some combinations becomes negative. The distinction between positive and negative birefringence is discussed below.

Figure 9 shows the birefringent response of the rigid and flexible materials. It should be noted that oscillograph traces resulting from the photocell-polariscope arrangement indicate light intensity, and do not distinguish between birefringent signs. Signs were determined by placing a diametrally loaded, stress-frozen disk next to tensile specimens and observing the resultant fringe pattern for increasing loads. Fringe patterns were observed through a small hand-held polariscope without disturbing the photocell output. The load axis of the disk was perpendicular to the tensile specimen. Using white light, it was observed that integral fringe orders in a flexible material cancelled the same fringe order of the disk. In other words, fringe order one in the tensile specimen produced a black fringe order one in the disk, while all other fringe orders in the disk remained colored. This was also true for successively increasing fringe orders. This characteristic of the flexible material was defined as negative birefringence. Leven's (30) standard mixture was used for the disk.

Fringe orders of the same order were not cancelled with the rigid material. Instead, equal fringe orders remained colored. Positive birefringence was assigned to this

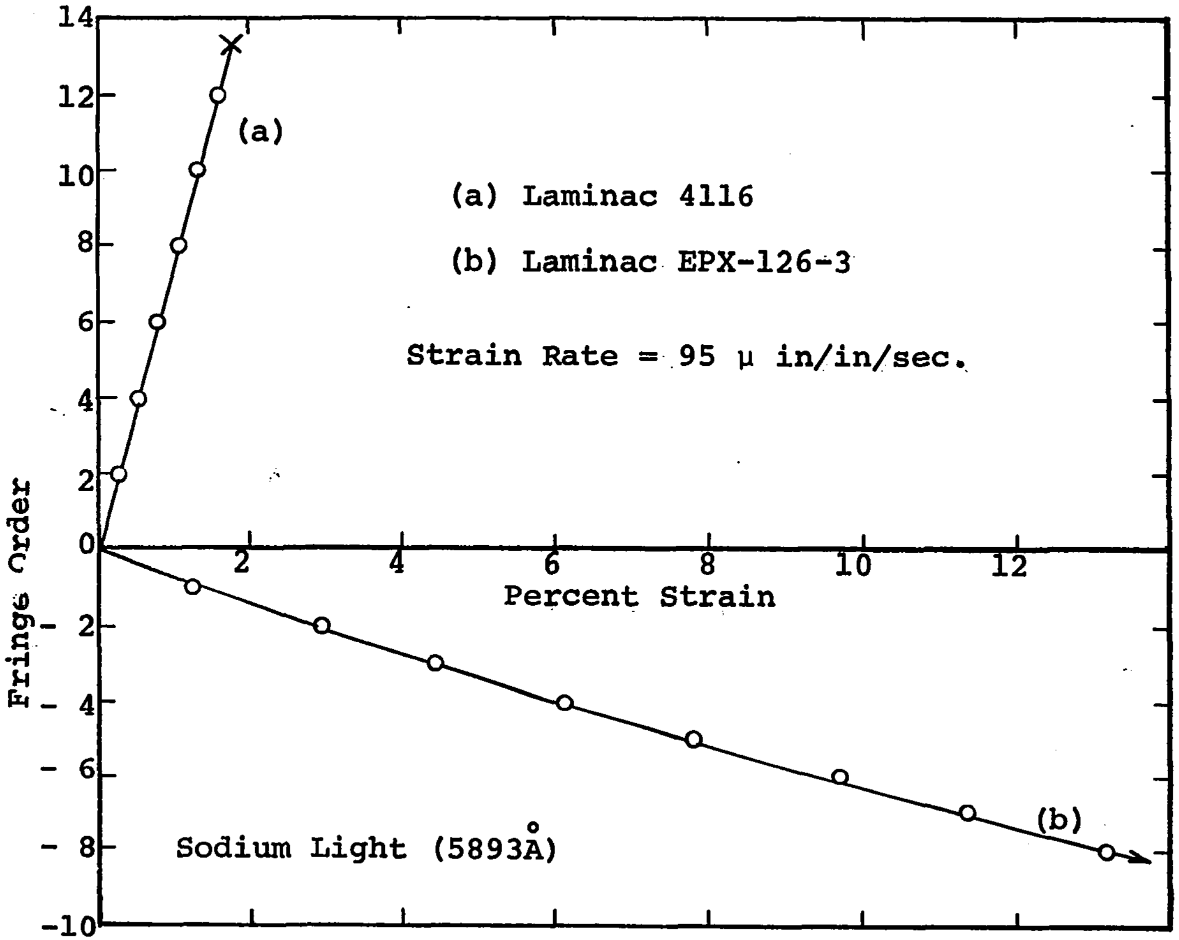

Fig. 9. Birefringence vs. axial strain for Laminac polyester resins

phenomenon.

The mechanical and optical properties of Laminac materials can be varied by changing the flexible/rigid resin ratio. The influence of resin ratio on mechanical properties should be considered first. Material properties must obey certain requirements in order that model results may be applied to a prototype. The first indication of the usefulness of model materials lies in stress-strain curves. In the present investigation it was desired to obtain a stress-strain curve that exhibited little or no strain hardening.

Stress-strain curves for several combinations of flexible and rigid resins are presented in Fig. 10. The curves show large strains that are desired, and a rise in stress with an increased amount of rigid resin. A 60:40 mixture of flexible and rigid resins satisfies the requirement stated above. Thus, this mixture was selected for further study.

Once the 60:40 mixture was selected it was desirable to study the effect of strain rate on material response. Figure 11 portrays the strain rate dependence. Stress-strain curves for the higher strain rates exhibit higher stress levels than those for the lower strain rates. An increase in rate also increases the modulus of elasticity.

The stress-strain curves depend on strain rate, but Fig. 11 indicates that there exists some minimal strain rate such that curves developed at this rate or less are the same.

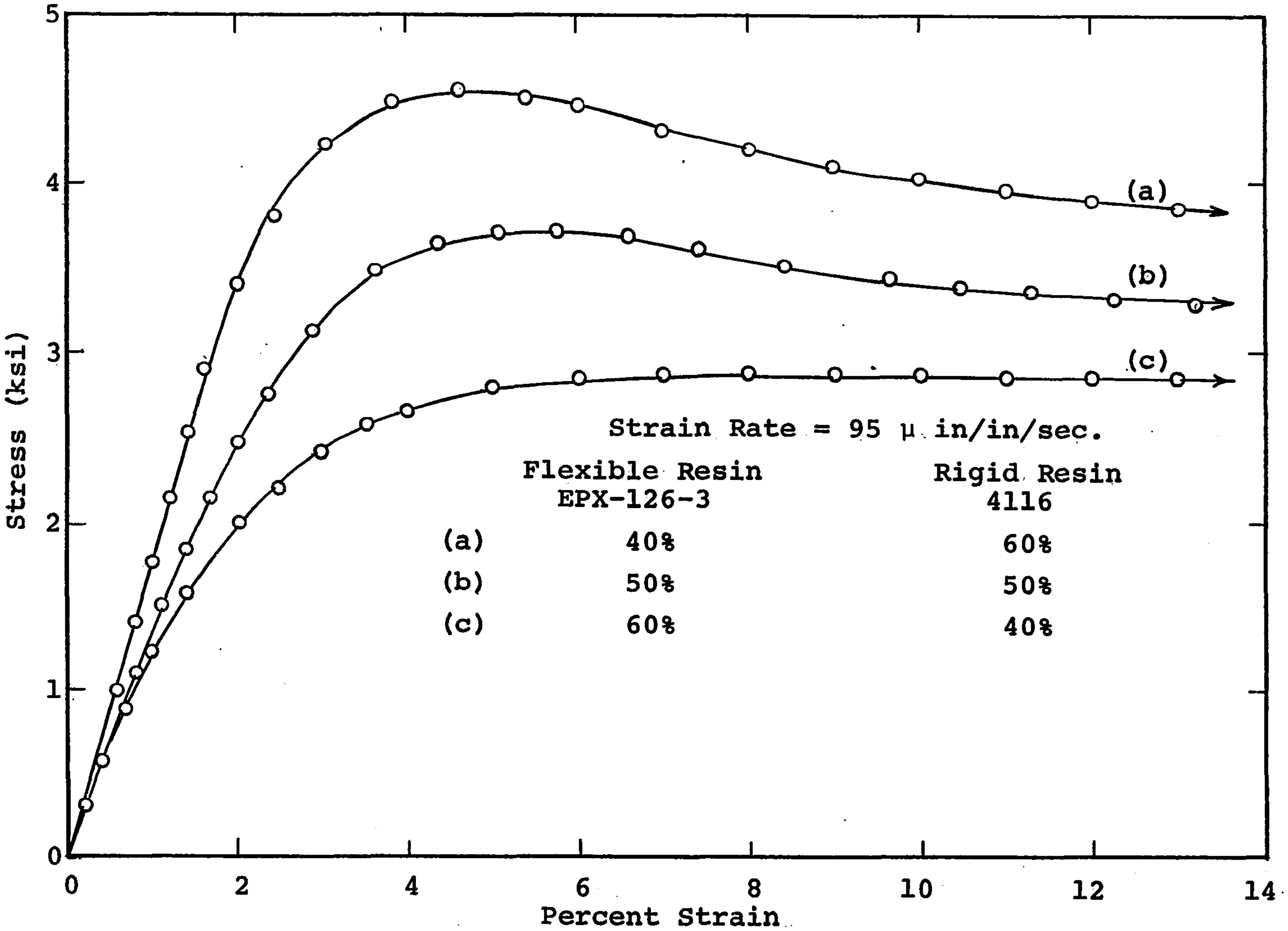

Fig. 10. Stress-strain curves for several combinations of flexible and rigid Laminac polyester resins

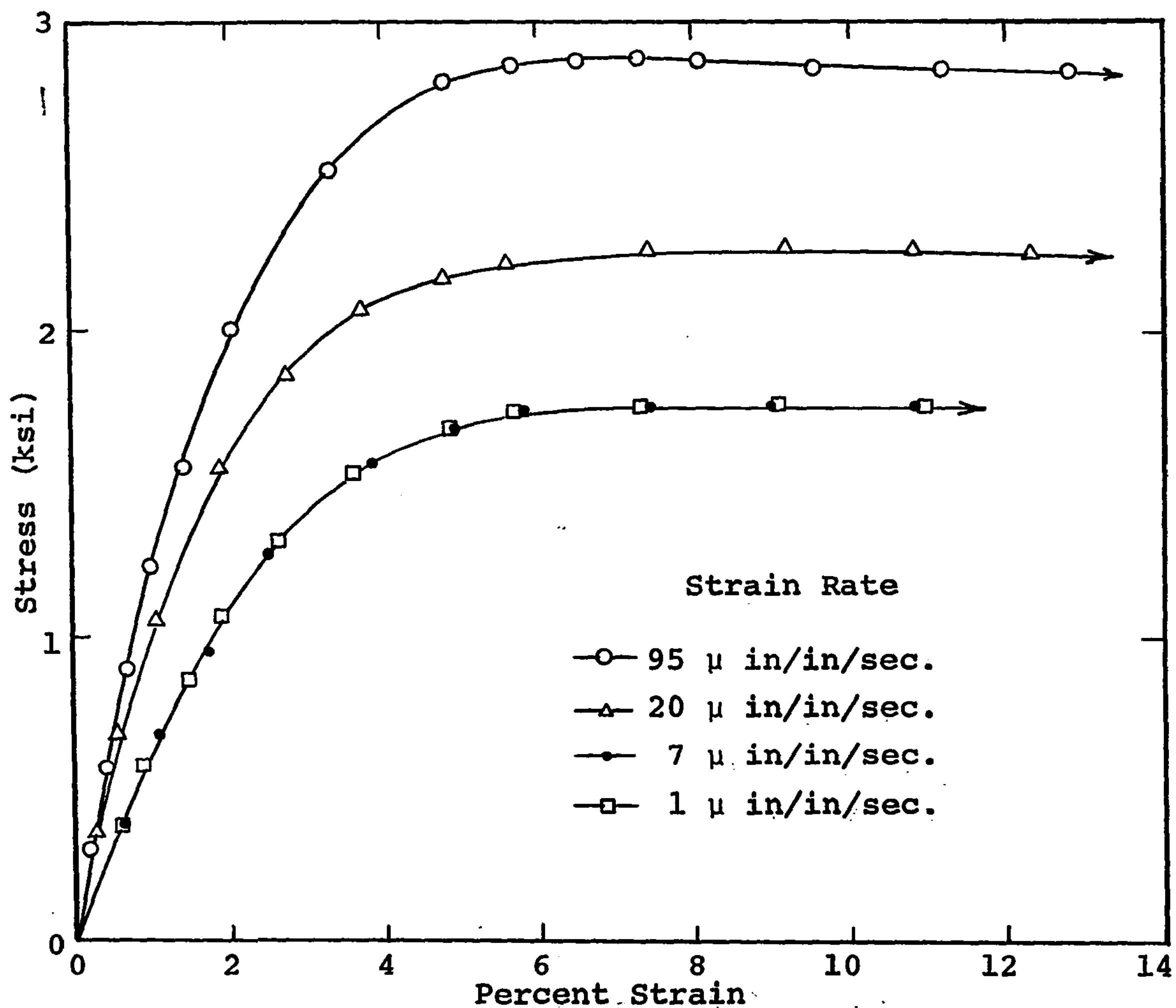

Fig. 11. Stress-strain curves as a function of strain rate for a 60:40 mixture of Laminac polyester resins

This strain rate independence is important since rate effects are usually neglected in the stress-strain diagrams of most structural metals at room temperature. Transition from model to prototype should be based on the lower curve.

Birefringence-stress results are shown in Fig. 12, and reveal the same strain rate independence as mentioned above.

Figure 13 demonstrates the strain rate dependence on birefringence-axial strain. Birefringence is initially positive, passes through zero, and becomes negative. Initial slopes of the curves are positive and rate dependent, but soon after reaching the maximum positive fringe order the slopes become negative and rate independent. The curves portray rate independence as previously discussed. Figure 13 also clearly indicates a multi-valued relationship between strain and birefringence. The state of strain is dependent on the history of loading, which, from a photomechanics point of view, is undesirable. Thus, at this stage it appears that Laminac polyester resins have good mechanical response but poor optical response.

Good mechanical properties prompted further study of the optical behavior. This study revealed a very important characteristic: upon unloading a tensile specimen the fringe order increased over that of a loaded specimen. Subjecting tensile specimens to different levels of strain, releasing the load, and counting fringe orders immediately upon release

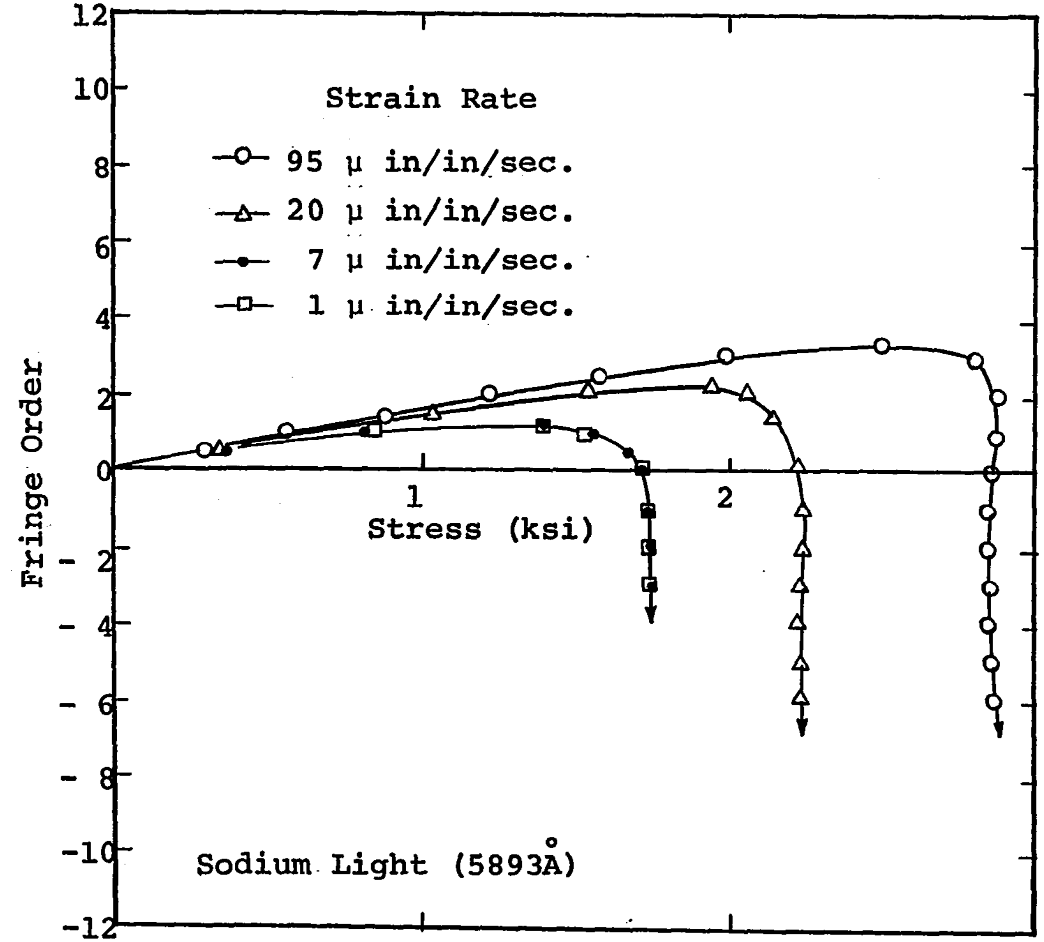

Fig. 12. Birefringence-stress as a function of strain rate for a 60:40 mixture of Laminac polyester resins

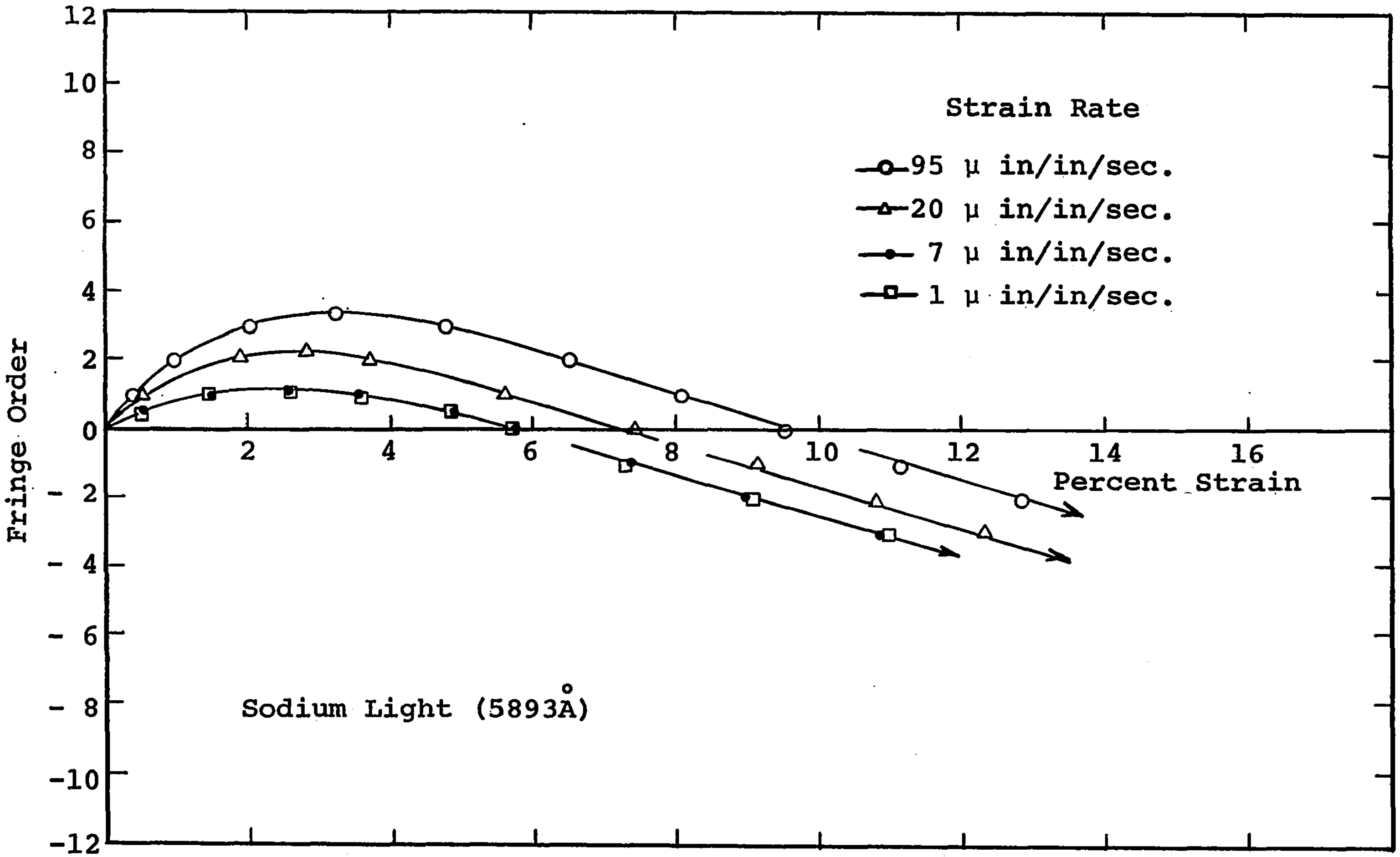

Fig. 13. Birefringence-strain as a function of strain rate for a 60:40 mixture of Laminac polyester resins

of load resulted in the straight (unload) line shown in Fig. 14. Unload birefringence (fringe order immediately upon release of load) was found to be negative using the disk method previously discussed. Thus for a uniaxial state of stress the strain may be determined by using the unload fringe order and the unload curve of Fig. 14.

It is interesting to note that the unload birefringence curve appears to be independent of strain rate, at least within the range of strain rates employed. However, this does not mean that testing can occur at any desired strain rate since material properties are rate dependent. Model to prototype transition requires use of the rate independent loading curve.

Unload birefringence decays with time. Since unloading a specimen and taking an isochromatic photograph takes a certain amount of time, it was necessary to determine the relationship between unload fringe order and time. The photocell-oscillograph arrangement was used to obtain fringe order-time data. The results are depicted in Fig. 15. Curves resulting from unloading at different levels of strain, but at the same loading strain rate, show a decrease of approximately 1/4 fringe order in 20 sec. Using this fringe order decay and the unload curve gives an indication of the error in strain that might be expected due to fringe order decay. Defining error in strain as $\Delta\varepsilon/\varepsilon$, and using the linear unload fringe

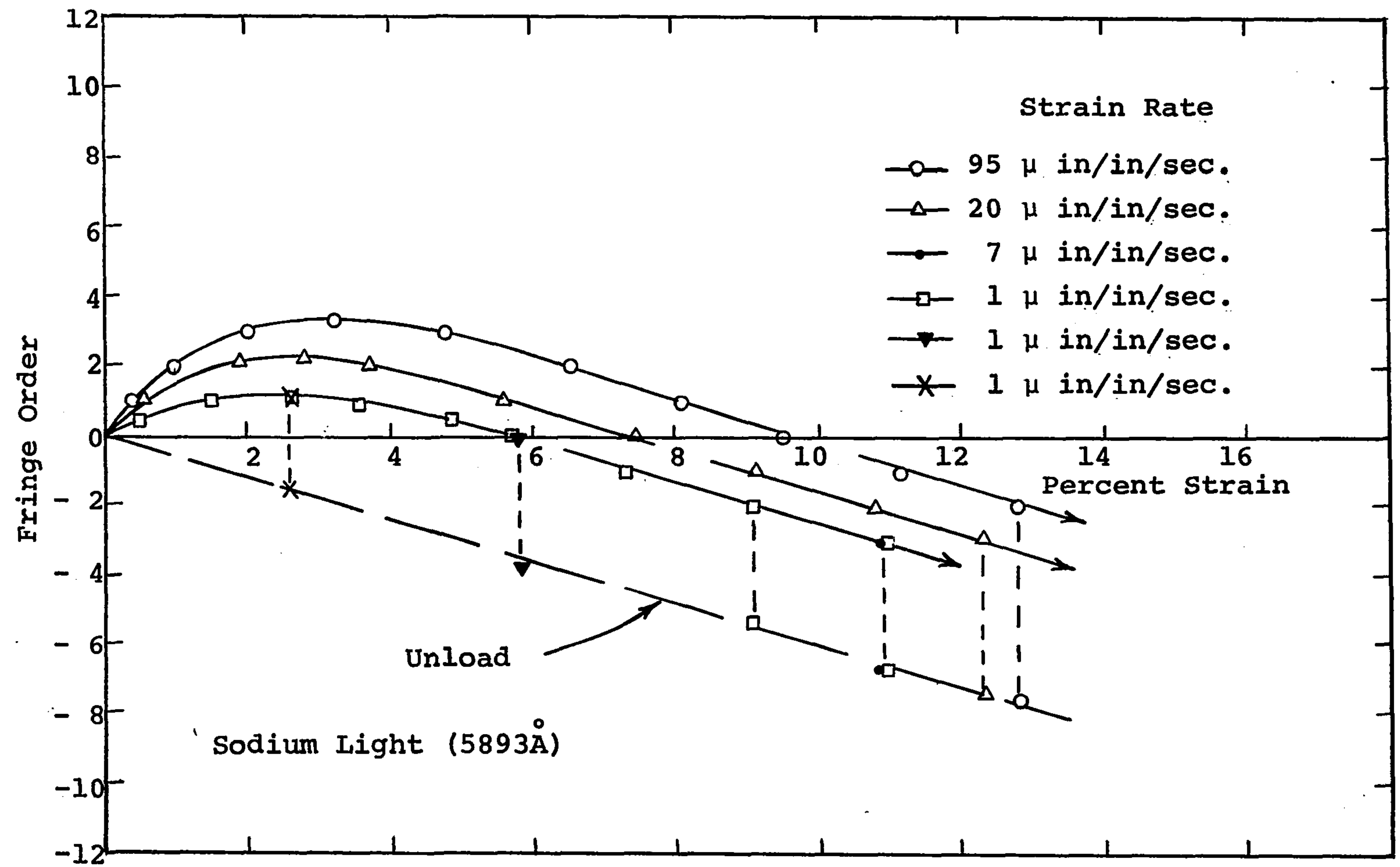

Fig. 14. Fringe order-strain for loading and immediately upon removal of load, 60:40 mixture of Laminac polyester resins

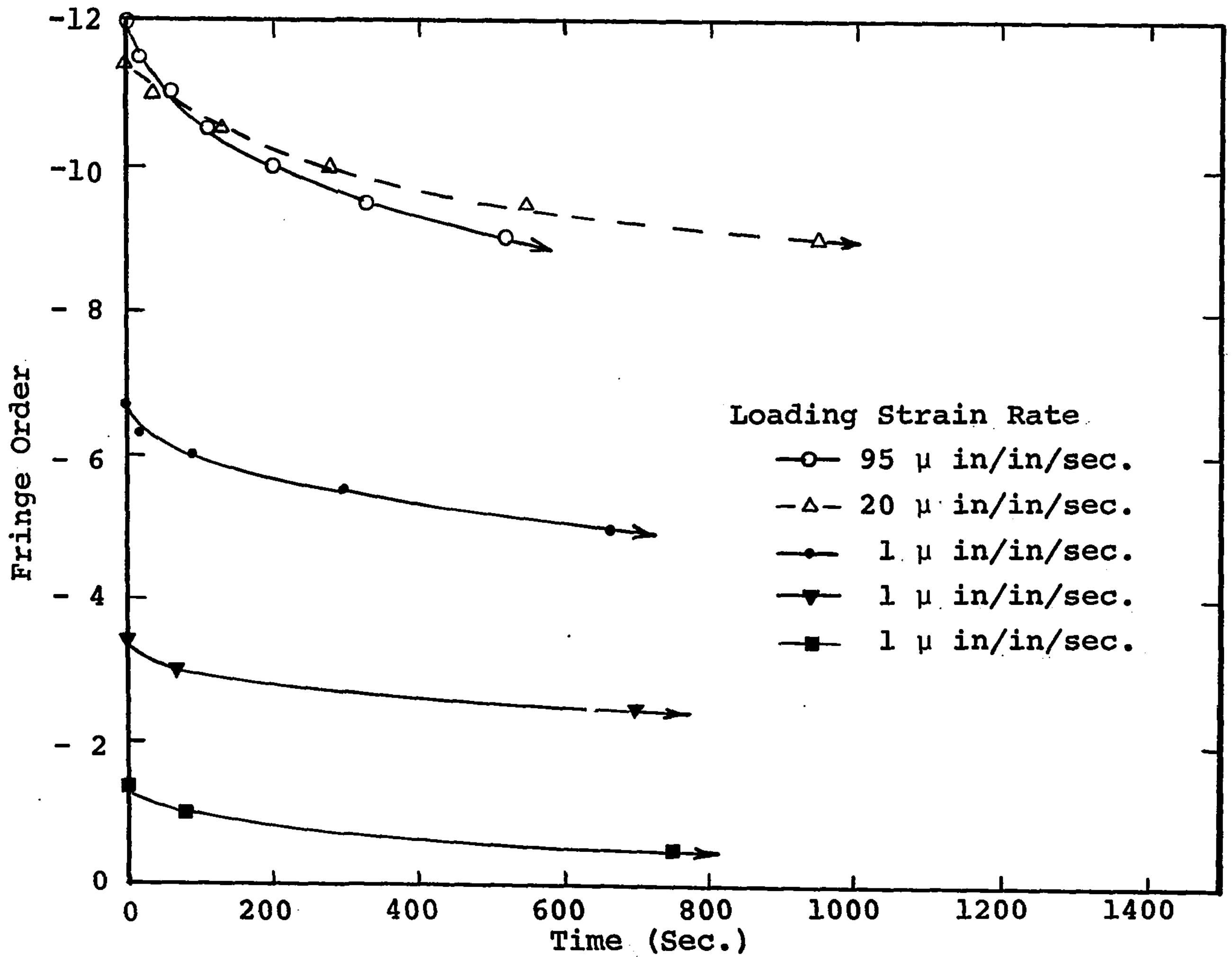

Fig. 15. Fringe order decay after removal of load, 60:40 mixture of Laminac polyester resins

order (N), results in an error of $\Delta\varepsilon/\varepsilon = \Delta N/N = 1/(4N)$. Therefore, greater errors can be expected for low fringe orders than for high fringe orders, which implies that great care must be exercised in recording data.

The 20 sec. time interval is more than ample to take isochromatic photographs. In fact, it was found that two people could unload a specimen and take light and dark field isochromatic photographs in about 10 sec.

Figure 15 also reveals that fringe decay at higher strain rates is initially more rapid than is the decay at lower rates.

Poisson's ratio was found from the ratio of transverse to longitudinal strain. Transverse strain was determined by means of a micrometer, and longitudinal strain was found in the manner previously discussed. The results are depicted in Fig. 16. Within the range of strain considered the material has a constant Poisson's ratio of 0.45.

C. Casting Technique

Numerous techniques exist for producing satisfactory castings of epoxy resins. As an example, refer to the work of Leven (30). The casting procedures for polyester resins are not so well known, at least in the way of published papers. Thus, it was necessary to formulate a method for casting high quality sheets of polyester. The procedure

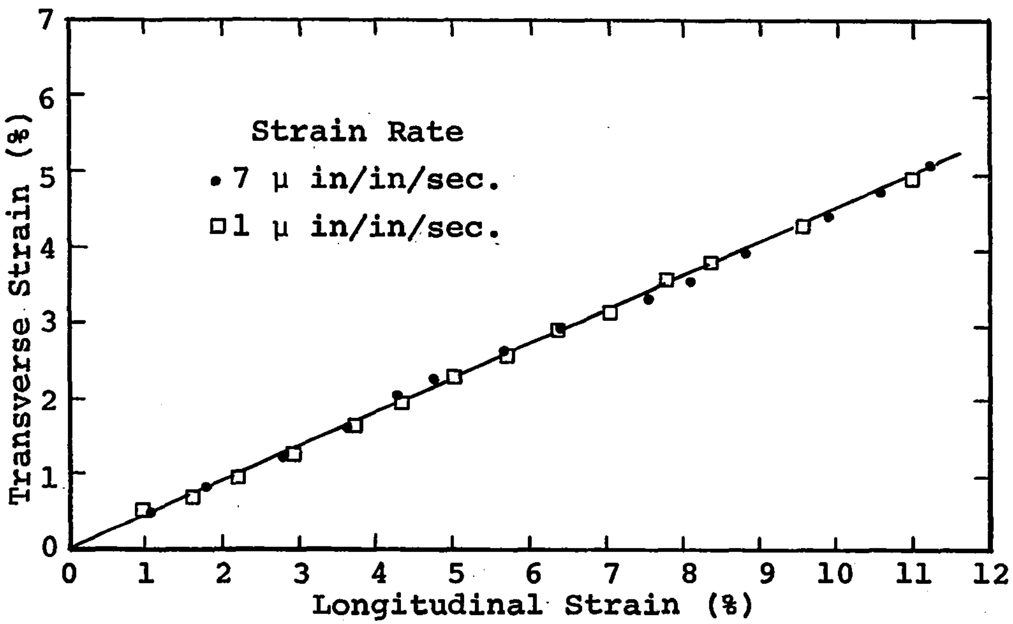

Fig. 16. Poisson's Ratio, 60:40 mixture of Laminac polyester resins

described below will also include several aspects of casting epoxies with the hope that other investigators will find the information helpful.

Vertical molds and open-faced horizontal molds were considered. The horizontal mold presented several problems; oven vibrations and fan currents tended to cause an irregular surface at the open-face, great care must be exercised in leveling the mold to insure sheets of uniform thickness, and the open-face of the polyester castings were tacky. Even though these problems could probably be solved, it was felt that vertical molds would eliminate most of the problems associated with horizontal molds.

Plate glass sheets were used for the two faces of the mold. Metal spacers placed between the glass plates controlled thickness, and neoprene rubber tubing placed inside the spacers formed a gasket between the glass plates. Binder clips held the plates together.

Before casting, the glass plates were carefully cleaned with acetone, then the plates and rubber tubing were coated with a mold release agent. Hysol AC-4-4368 and Dow Corning 20 Compound were found suitable for the epoxies, while ReleasaGen H-15-1 was used for the polyesters. Other release agents considered for polyester castings were silicones (such as those used for the epoxies) and ordinary paste wax. Use of these agents resulted in castings with poor surface finish due to partial sticking to the molds. Further experimentation with these release agents might result in good polyester castings.

Both the epoxies and polyesters investigated gelled at room temperature, thus great care had to be exercised when mixing the resin components and hardeners. At room temperature the viscosity of the resin components is such that air bubbles are trapped in the mixture while stirring, resulting in a poor casting unless the bubbles are removed. Two approaches were used to eliminate air bubbles.

First, the mixture was placed in a vacuum chamber, resulting in removal of the entrained air after a length of time. For some mixtures of resin and hardeners the time was such that the mixture gelled before all the bubbles were gone. Careful observation of the bubble formation indicated that a short interval of time existed in which the bubbles appeared to be gone, but then began to form again. (This observation was more pronounced for the polyesters than for the epoxies.) The new formation was believed due to removal of some constituent of the mixture. Removal of the vacuum between bubble formations left a mixture that was essentially bubble free; epoxy mixtures had some entrained air remaining, while the polyester mixtures were almost completely bubble free. This method of solving the air bubble problem resulted in good castings. Nevertheless, this method was dropped in favor of the second method, which gave comparable, or better results.

The second method did not rely on a vacuum system. Very careful mixing and slow stirring (about 10-15 minutes) gave a mixture that was almost free of entrained air. The pot life of all mixtures was longer than the stirring time. The high viscosity of the epoxies did not allow air bubbles to float off before solidification, whereas the low viscosity of the polyesters resulted in bubble free castings.

All mixtures were slowly poured into the molds through a funnel. No additional air bubbles were created in the pouring process, provided the pouring was slow.

The mixtures were then subjected to the following thermal cycles:

> Ciba 502-Lancast A; room temperature cure for 24 hrs. and post cure at $80^{\circ}C$ for 16 hrs.
>
> Paraplex and Laminac polyester resins; room temperature cure for 24 hrs. and post cure at $80^{\circ}C$ for 16 hrs.

Great care must be exercised in removing the casting from the mold. Careful prying apart of the mold faces released the epoxy castings, but was futile for polyesters. Polyester removal consisted of placing a sharpened hack saw blade between the casting and glass plate, and lightly tapping the blade as it was moved along the casting. Forcing a razor blade under the sheet and careful lifting by hand removed the casting from the second plate of glass.

IV. APPLICATIONS

At this stage of the investigation it was desirable to study the feasibility of using the proposed technique to predict prototype behavior. Thus far it is possible to investigate stresses and strains in one-dimensional stress fields. Of particular importance is the study of stress concentrations at geometrical discontinuities.

As previously stated, Frocht and Thomson (14) list several conditions that must be satisfied in order that model results may be applied to a prototype made of a different material. Similarity of geometry poses no great problem, but the other similitude requirements are more difficult to meet.

One requirement is that the value of Poisson's ratio in the plastic range be the same for model and prototype. For the 60:40 mixture of Laminac polyester resins Poisson's ratio was found to be 0.45. A value of 0.5 is usually assumed for metals undergoing plastic flow.

The model and prototype materials must have the same law of yielding. This requirement was not investigated, as it would require a study in itself. Nevertheless, the study was continued, and it was found that good agreement was obtained between model and prototype, whatever the yield criterion may be.

Similarity of loading (magnitude and loading path) must exist. For metallic prototypes it is usually assumed that time effects are negligible. Elimination of time effects in the proposed material requires a very slow loading rate. In addition, the conditions in a calibration member (tensile specimen) are not the same as in a general problem. Loading a model in such a manner that the rate independent calibration curves are valid should give a reliable means of calculating model strains.

Model and prototype must have the same shape of the stress-strain curves, for loading and unloading. This investigation considered problems where no unloading took place, requiring study of stress-strain curves for loading only. As previously mentioned, a comparison between stress-strain curves can be made by means of the Ramberg-Osgood equation.

It was desirable to compare results using the proposed technique with those of theoretical and other experimental methods. Both theoretical and experimental stress concentration factors have been found for a plate with a central circular hole, loaded in uniaxial tension. The theoretical solution considered several Ramberg-Osgood curves, while experimental results are for aluminum alloys.

Budiansky and Vidensek (31) consider the problem of finding stresses in the plastic range around a circular hole in an infinite plate subjected to uniaxial tension. Their material is assumed to obey the simple deformation theory of plasticity. A plane stress solution is obtained by application of a variational principle in conjunction with the Rayleigh-Ritz procedure. The material is assumed to obey the Mises yield criterion and to have a Ramberg-Osgood stress-strain curve. Results are given for four different materials.

Budiansky and Vidensek assume a plate of infinite width. In order to approximate their plate a specimen with a width to hole diameter ratio of 13.3 was used, Fig. 17. Loading the specimen through 1/4"-thick steel plates clamped at the ends gave a uniform tensile loading. All tests were conducted at an applied strain rate of 1 μ in/in/sec.

The theoretical solution depends on the Ramberg-Osgood equation. Thus it was necessary to determine the value of n that gave the best fit to experimental data. Figure 18 shows the curve for n = 19, which gave the best fit for the 60:40 mixture of Laminac polyester resins.

The curves of Fig. 19 were used to determine stress and strain from birefringence measurements. Light and dark field isochromatic photographs taken immediately upon unloading gave fringe orders at the point of maximum stress. Average stresses were found from the known load applied to the test plate, and the gross cross sectional area. All stress concentration factors reported are based on average stress. These factors were found for six degrees of plastic distortion, for plates 1/4"-thick.

Typical dark field isochromatic photographs taken before

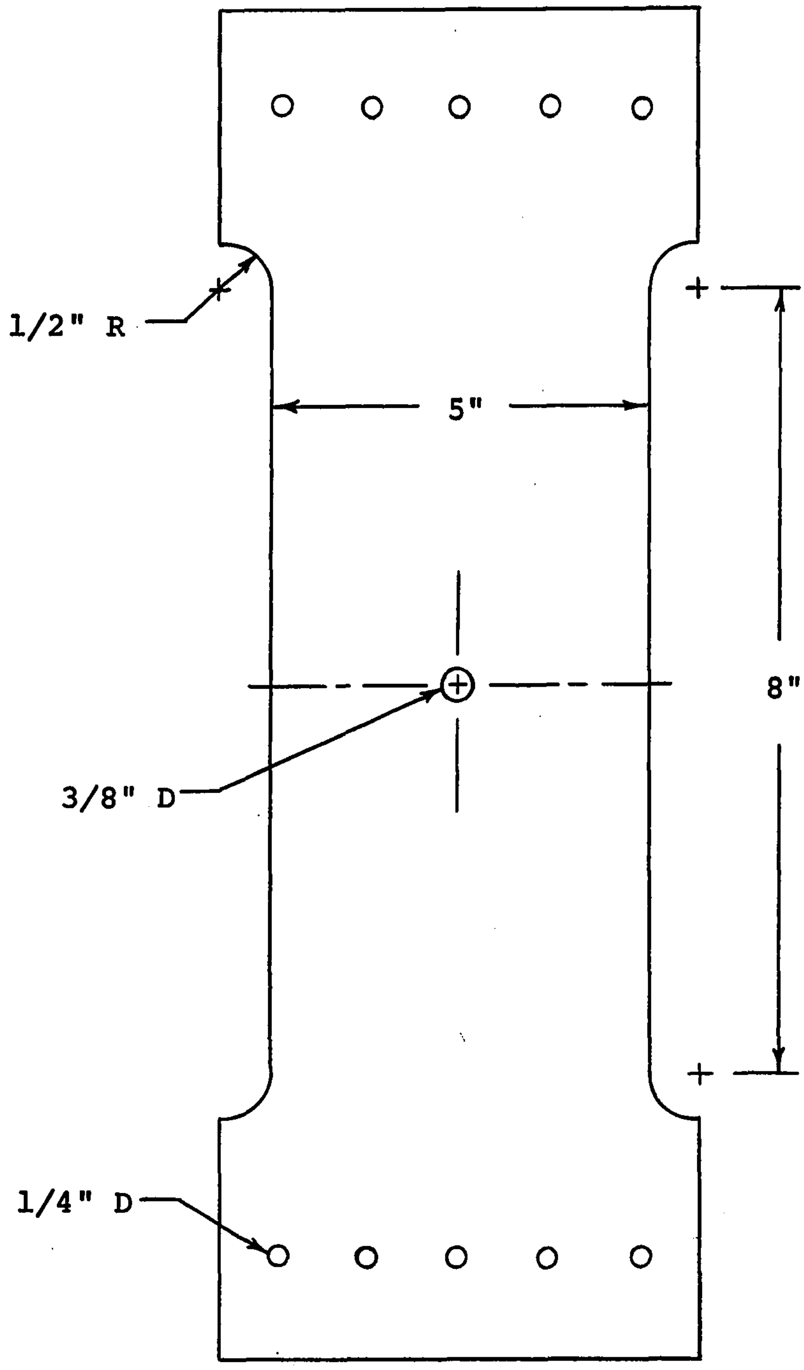

Fig. 17. Plate specimen

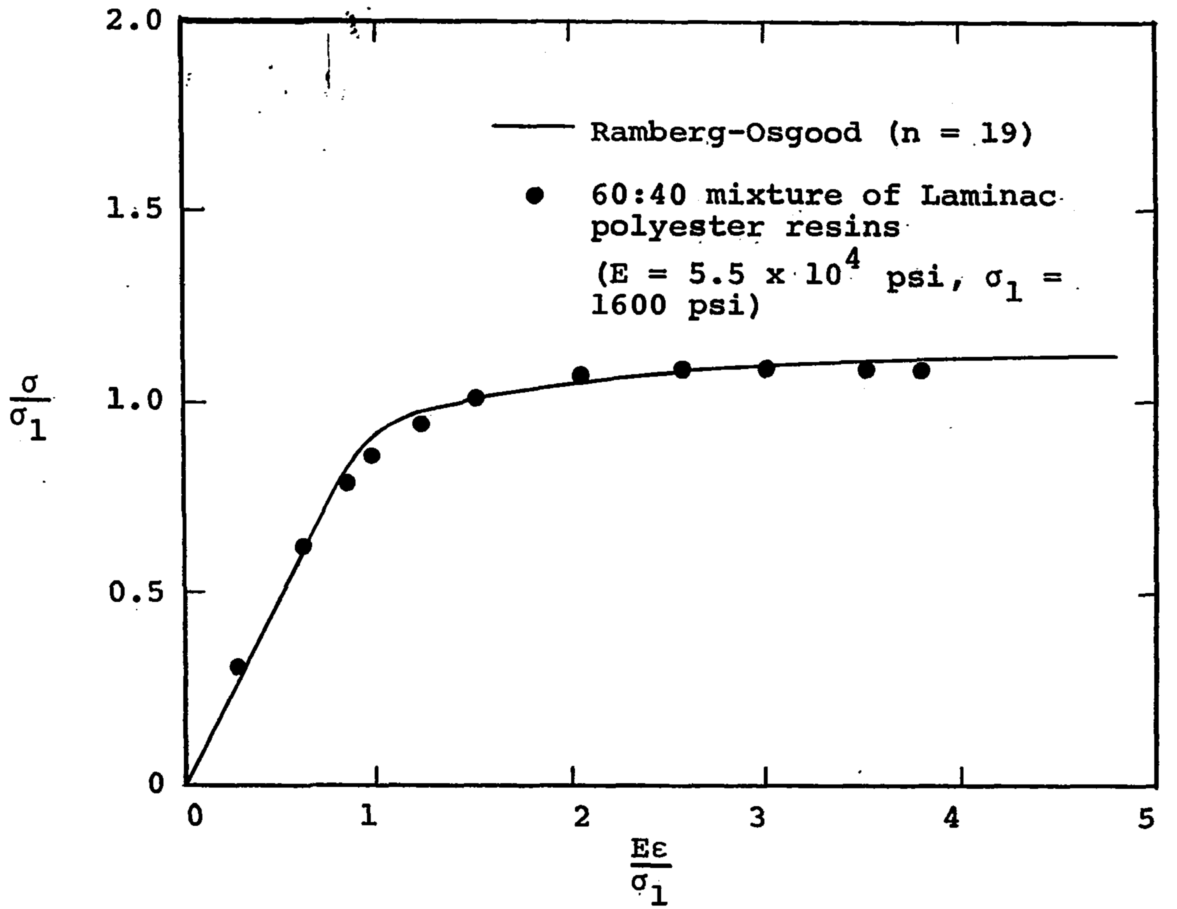

Fig. 18. Dimensionless stress-strain comparison

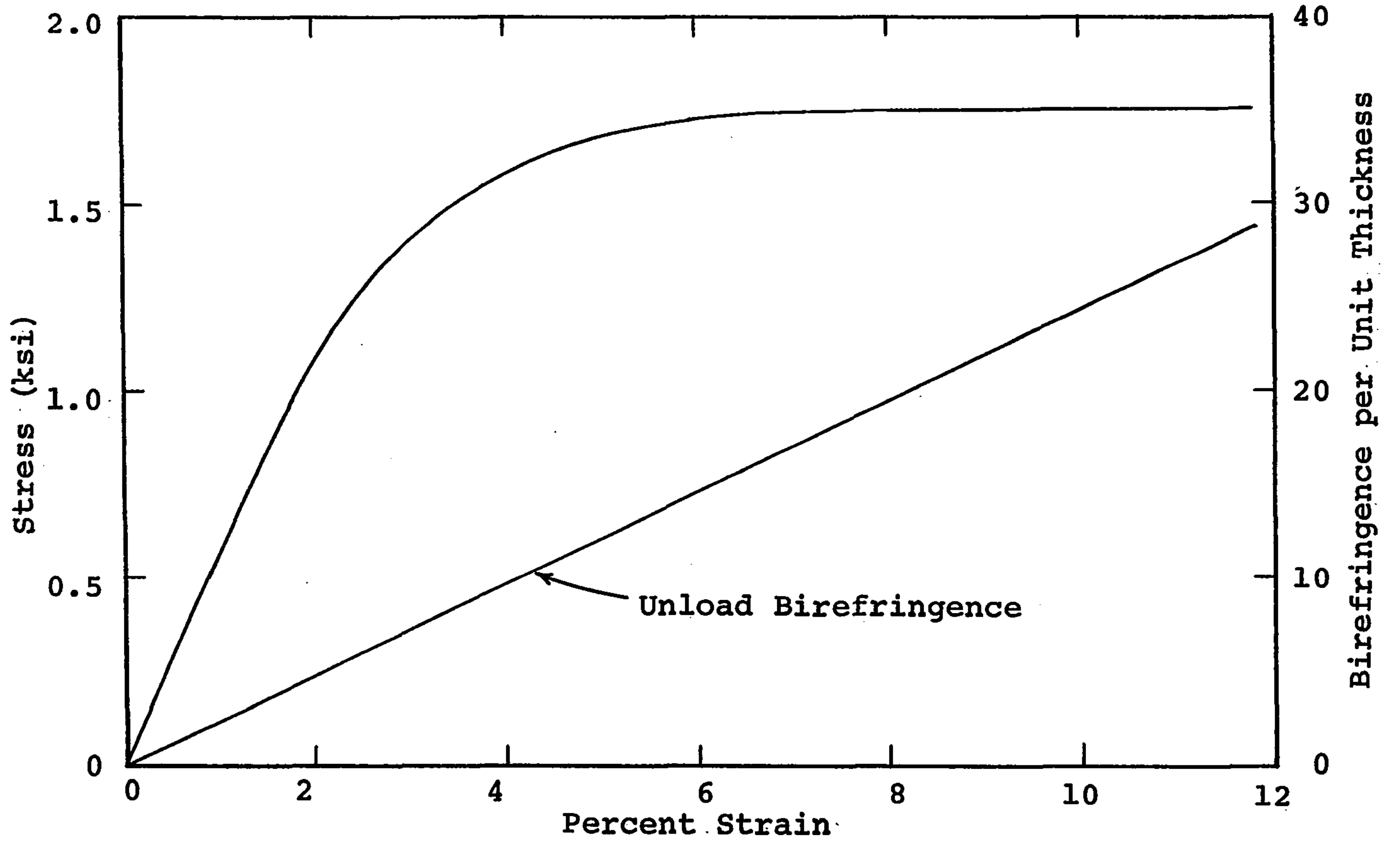

Fig. 19. Curves to determine stress and strain from birefringence measurements

and after unloading are shown in Figs. 20 and 21, respectively. The plate was loaded to a value of 1630 lb, resulting in large plastic distortion. Fringe orders are easily countable, and were even more so when the photographs were enlarged for analysis.

Figure 22 shows a comparison between theoretical and experimental stress concentration factors. There is good agreement between the results, the maximum deviation being about 8%. It should be mentioned that the maximum deviation occurred for $\lambda = \frac{\sigma_{avg}}{\sigma_1} = 0.83$, which gives a strain that lies outside the range of the calibration curves. To obtain this point it was assumed that the calibration (stress-strain and strain-birefringence) extended in the manner illustrated in Fig. 19.

Box (32) presents the results of an experimental investigation on non-alclad 24S-T aluminum alloy which was conducted to determine the variation of stress concentration factors with plastic strains. The test specimens were 4.5 in. wide with 1.0 in. diameter holes. Strain measurements of a grid were made with an optical slide comparator.

The dimensionless stress-strain comparison of Fig. 23 indicates that the polyester polymer can be used to compare results with those of Box. A stress concentration factor comparison is shown in Fig. 24.

Griffith (33) investigated uniformly dimensioned 24S-T

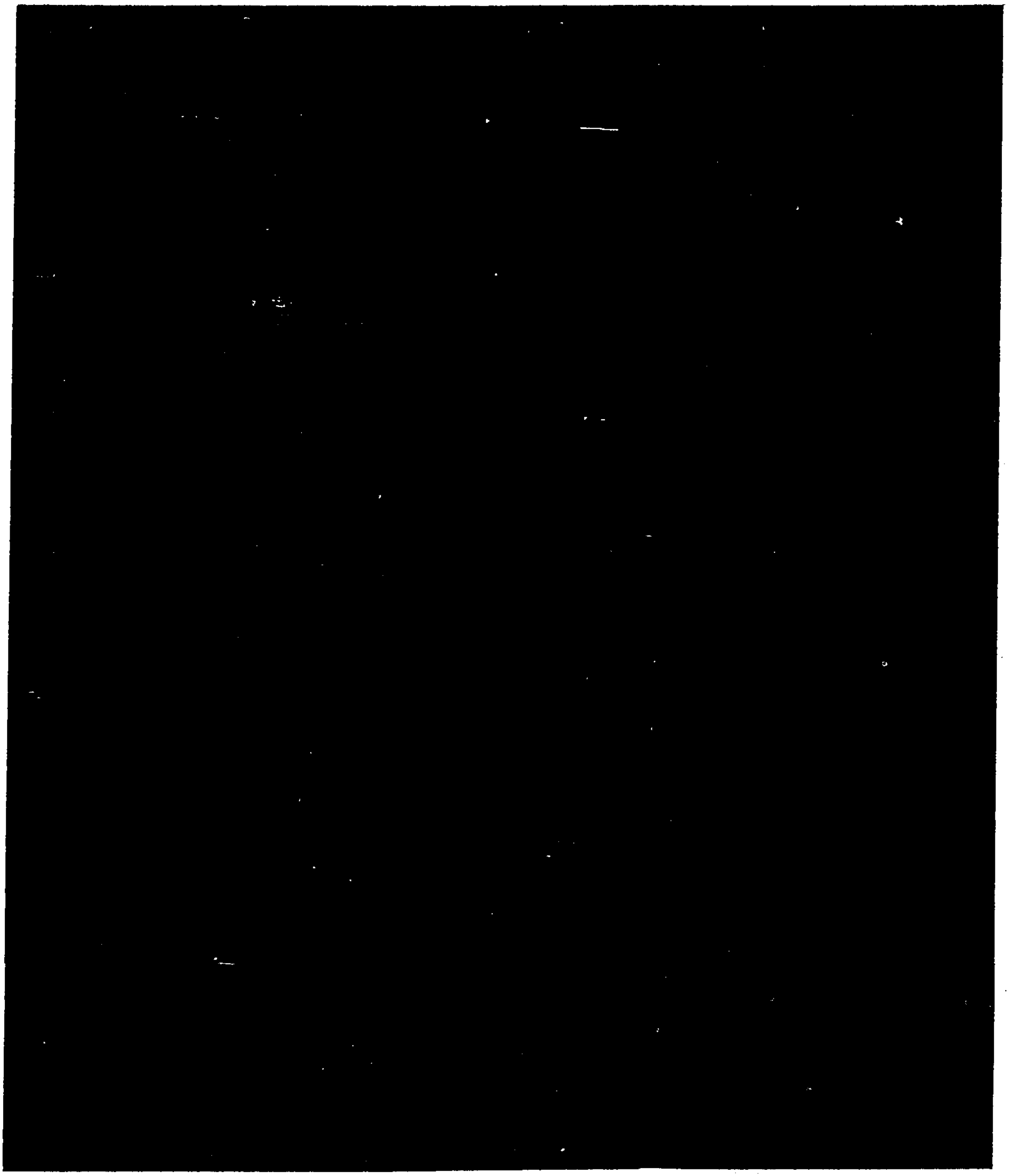

Fig. 20. Dark field isochromatic photograph before removal of load

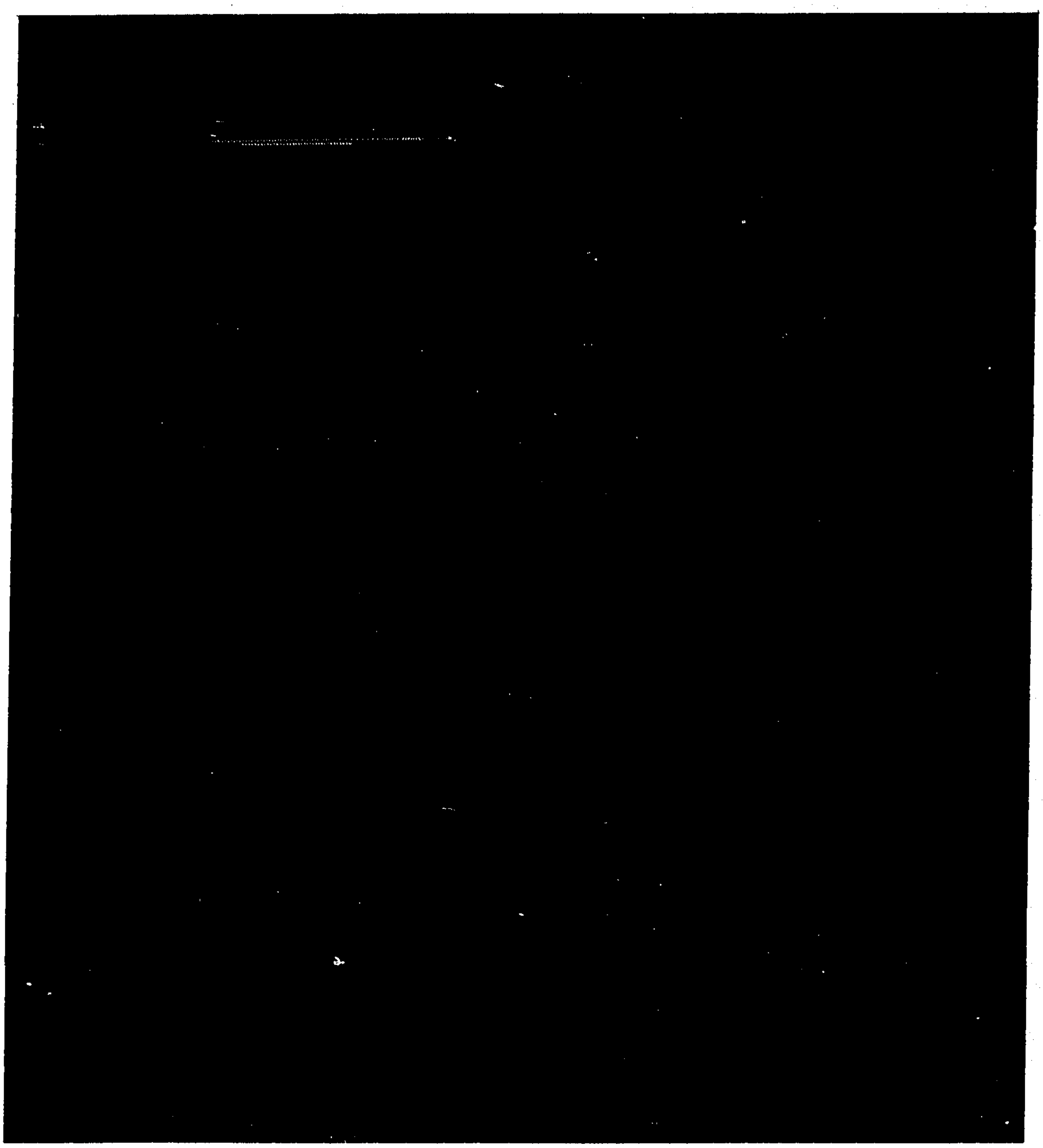

Fig. 21. Dark field isochromatic photograph immediately after removal of load

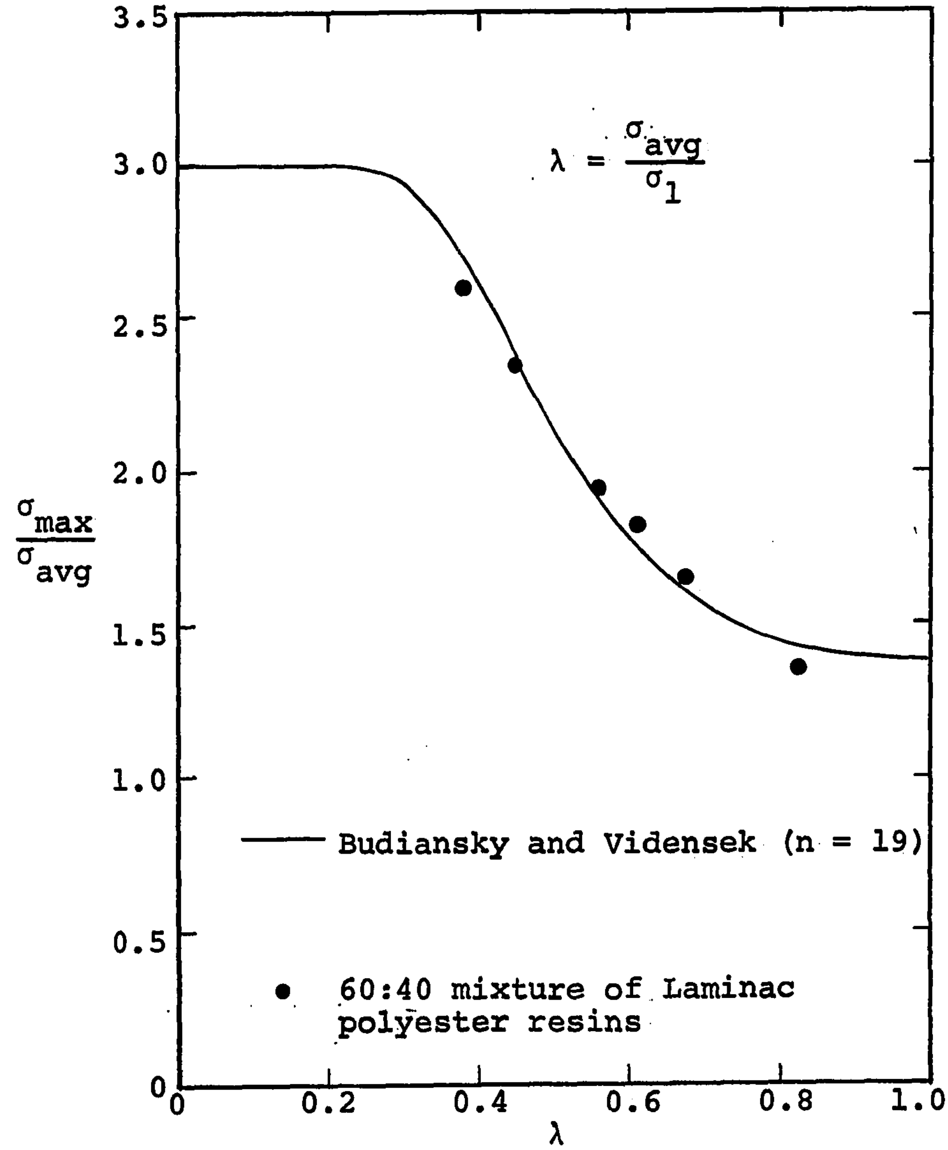

Fig. 22. Stress concentration factor comparison

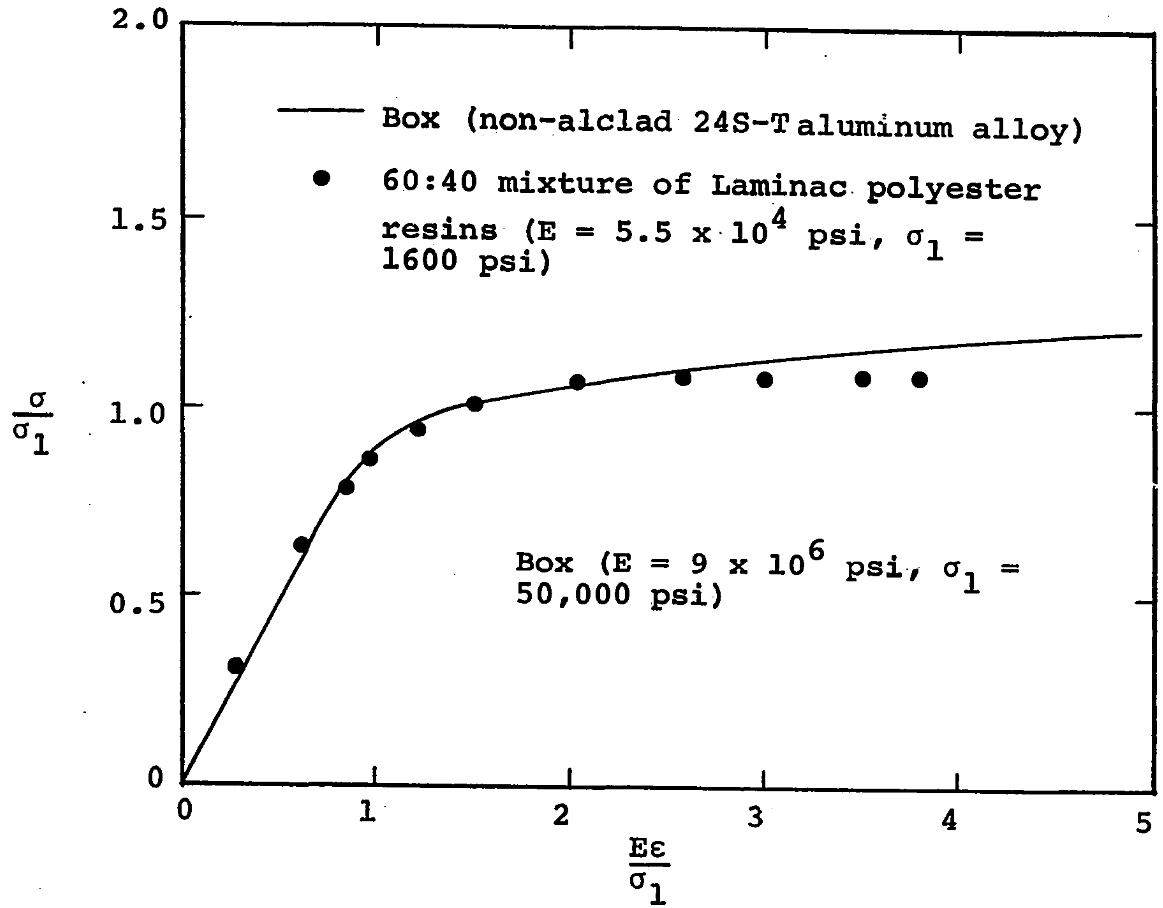

Fig. 23. Dimensionless stress-strain comparison

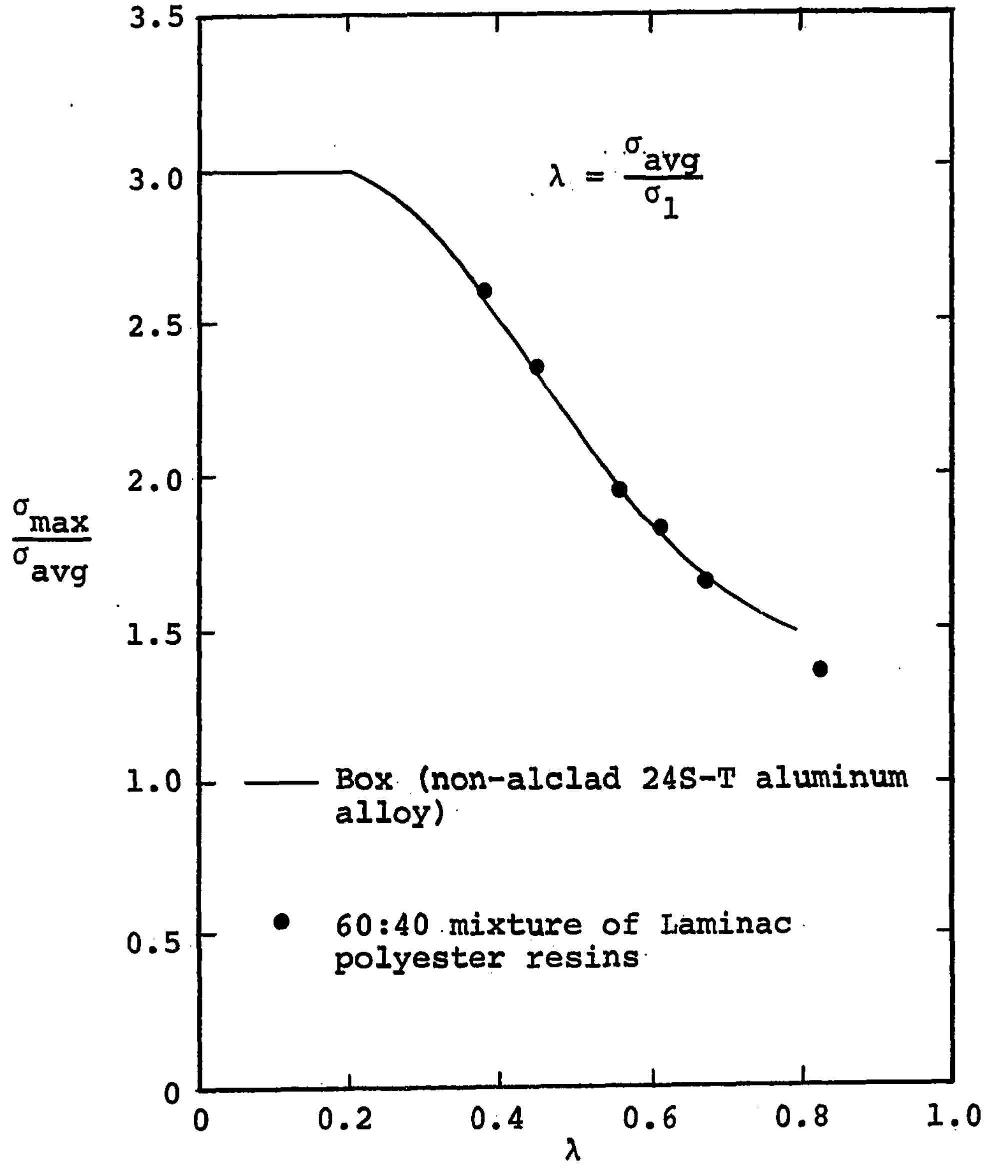

Fig. 24. Stress concentration factor comparison

aluminum panels with central circular holes. The test specimens consisted of panels 24 in. wide, with holes 4 in. in diameter. Electromagnetic and electric wire gages were used to measure strains.

Dimensionless stress-strain and stress concentration factor comparisons are portrayed in Figs. 25 and 26. Only fair agreement in stress-strain comparisons is reflected in the stress concentration factor results.

Durelli and Sciammarella (34) experimentally determined elastoplastic stress and strain distributions around a circular hole in an aluminum plate subjected to a unidimensional load. Their plates were 8 in. wide with 1.25 in. diameter holes drilled at the center. The moiré method was used to measure strains, and stresses were computed by means of the Prandtl-Reuss stress-strain relationships.

There is good agreement between results, as shown by Figs. 27 and 28.

The variation of the strain concentration factor as a function of λ can also be compared with the results of Box, Griffith, and Durelli and Sciammarella. This comparison is shown in Fig. 29. Agreement is good at low values of λ, but the difference tends to increase for increasing plastic flow. A comparison of stress concentration factors as a function of λ indicates that better agreement is obtained than is shown in Fig. 29. This discrepancy is probably due in part to the

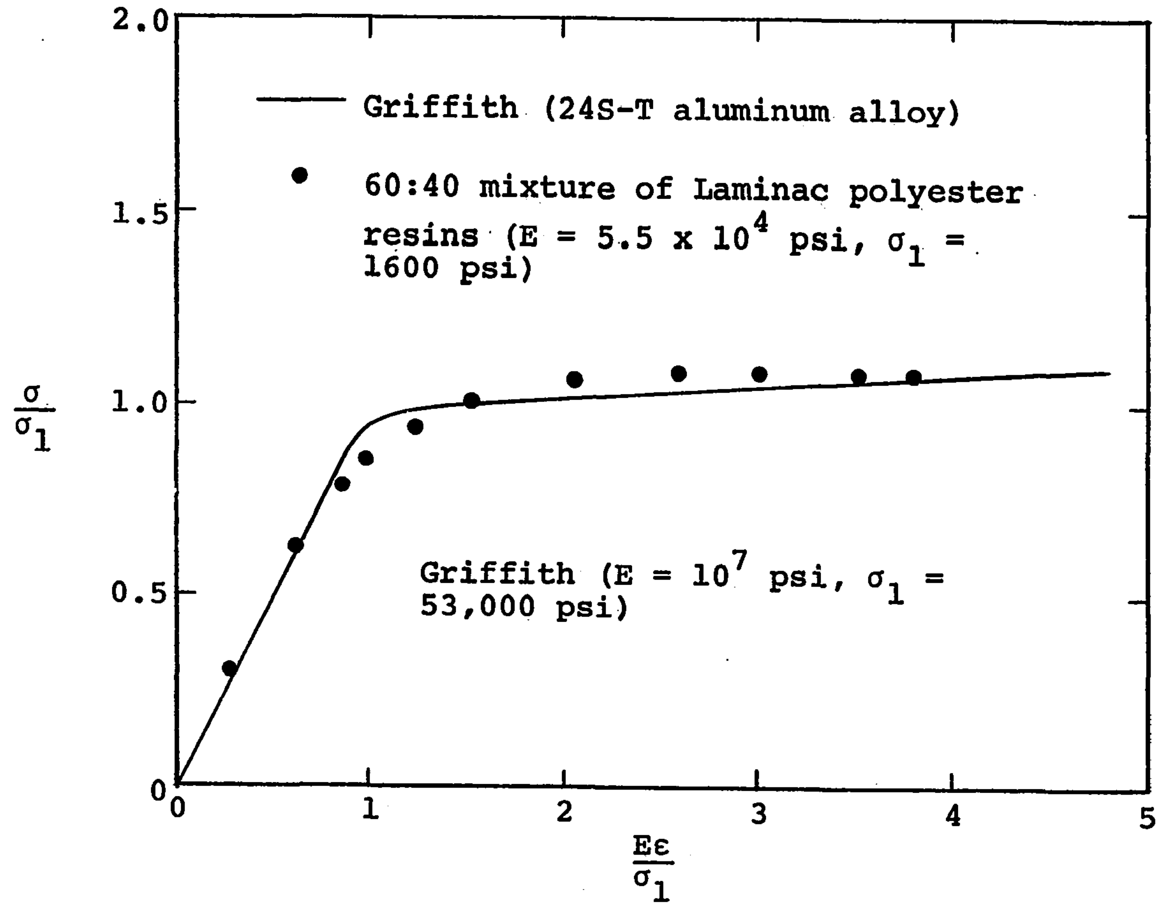

Fig. 25. Dimensionless stress-strain comparison

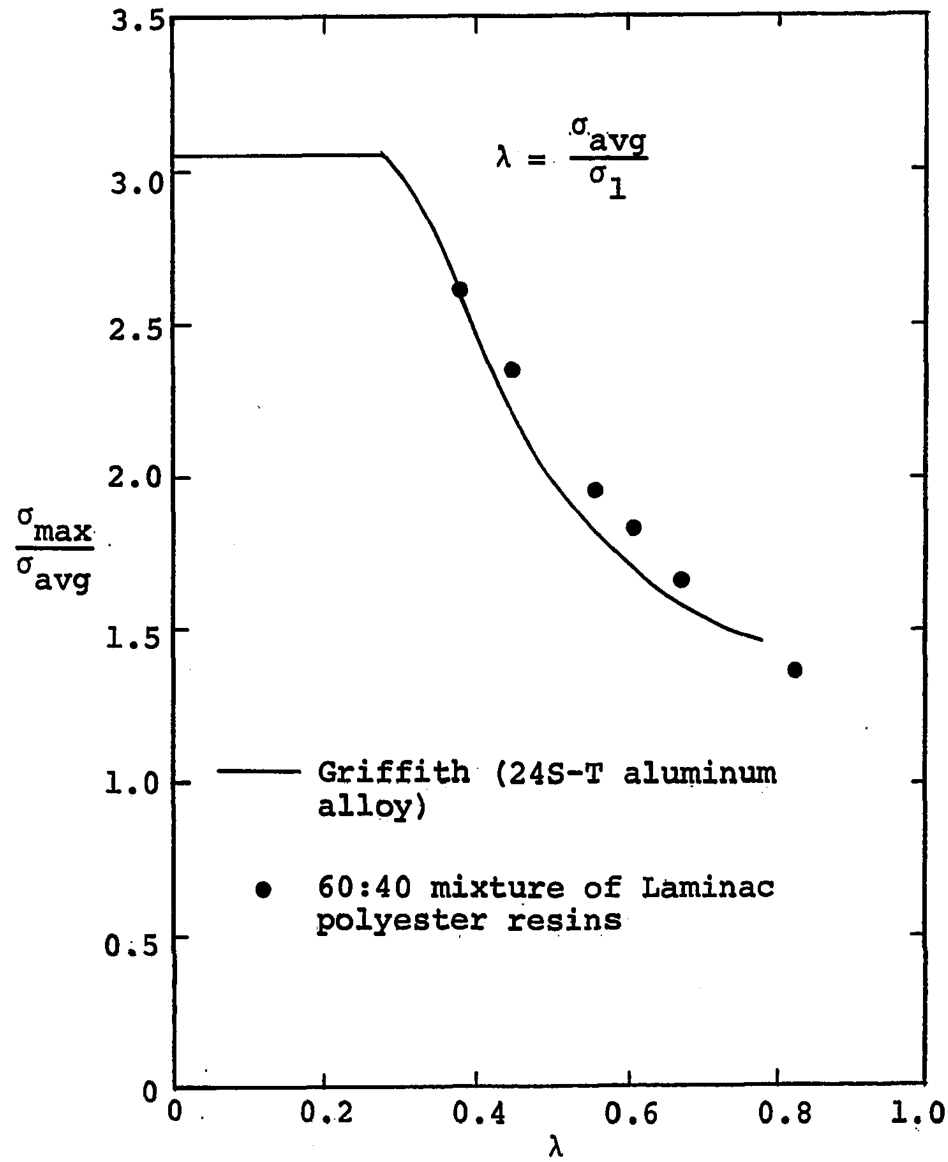

Fig. 26. Stress concentration factor comparison

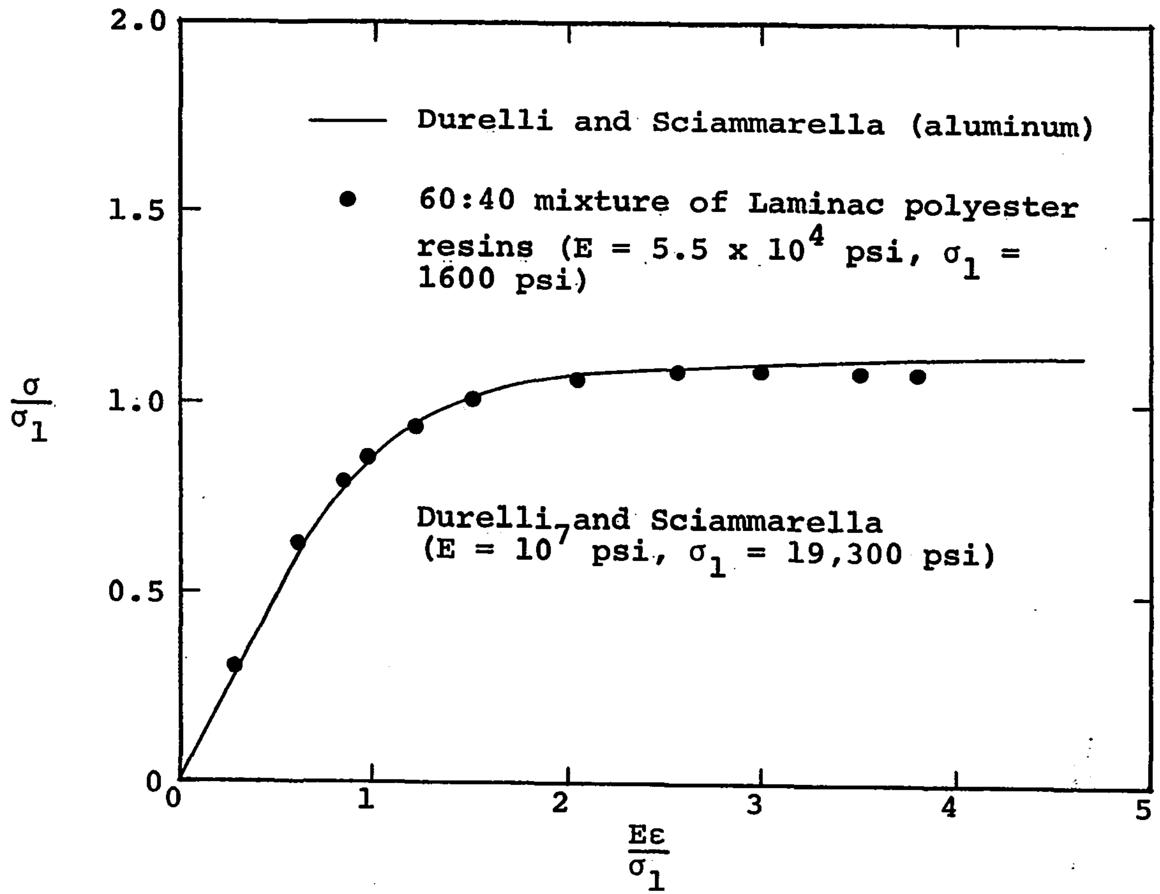

Fig. 27. Dimensionless stress-strain comparison

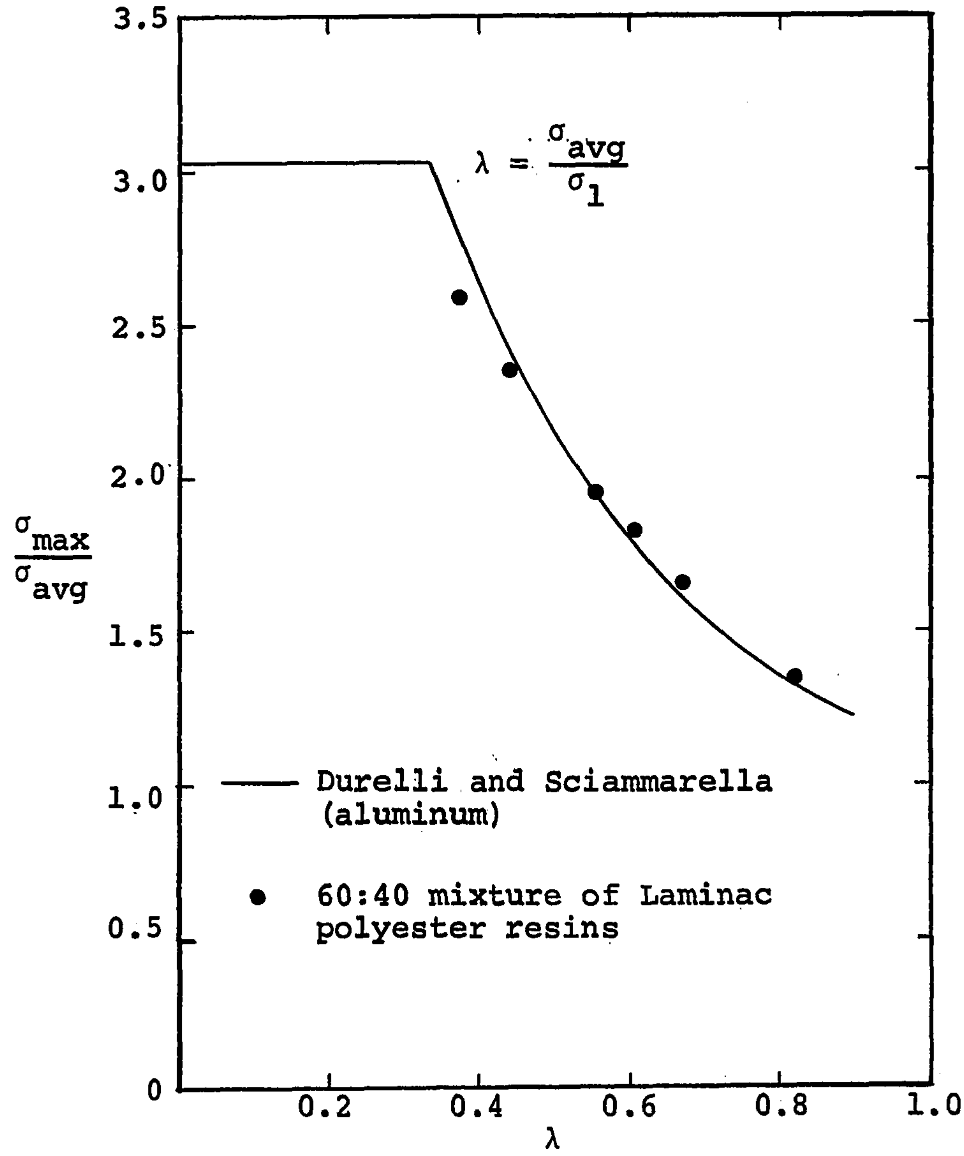

Fig. 28. Stress concentration factor comparison

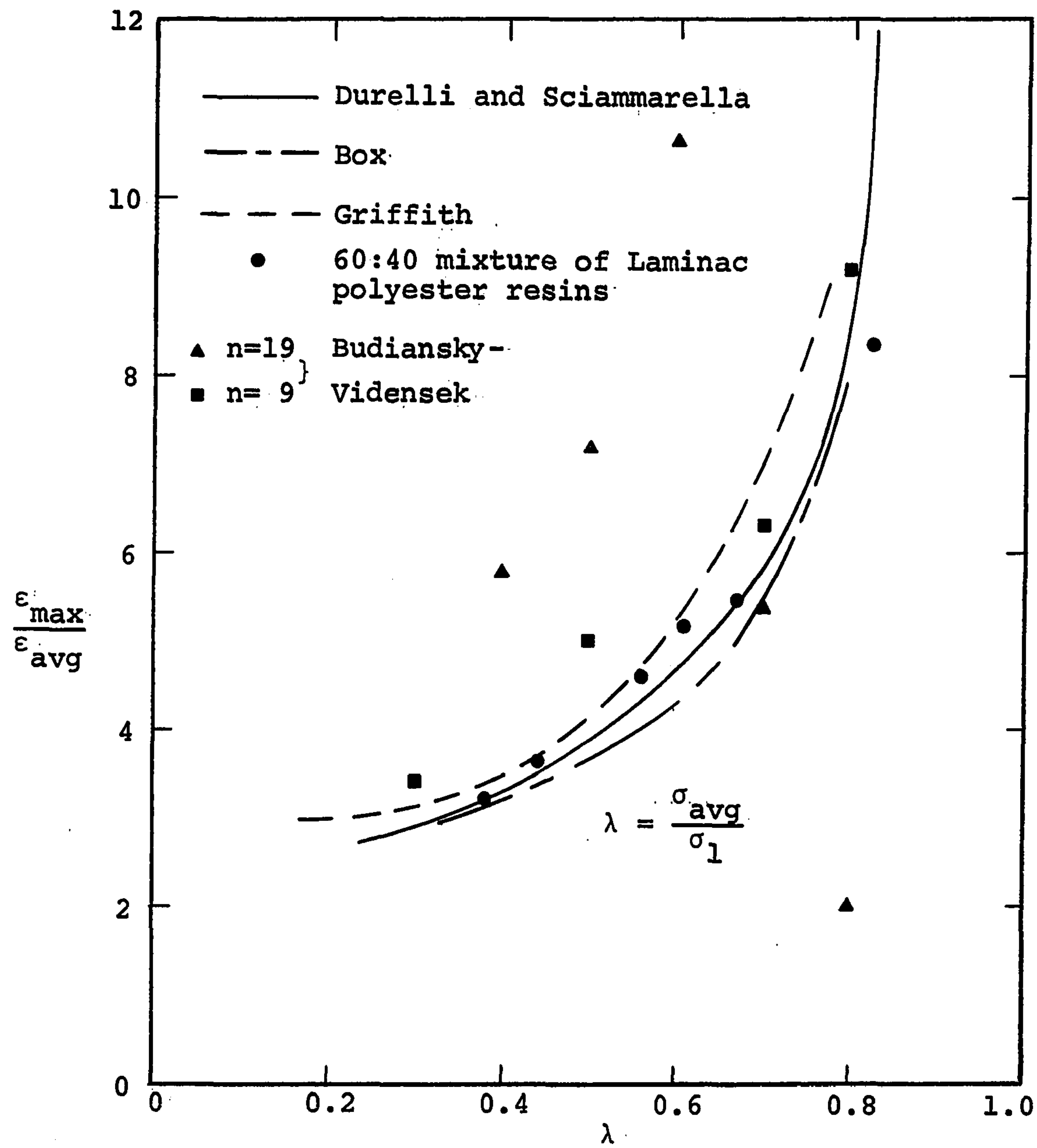

Fig. 29. Comparison of strain concentration factors

different testing methods, and in part to the shape of the stress-strain curve, which permits a small error in stress to be magnified in strain.

It is also possible to compare experimental strain concentration factors with theoretical values obtained from the Budiansky-Vidensek (31) solution. These results are also shown in Fig. 29. It is evident that very poor agreement exists between experimental and theoretical results. The theoretical solution for n = 19 shows a decrease in strain concentration factor with increasing load. This implies unloading which, according to Budiansky and Vidensek, invalidates the use of the stress-strain relations of the simple deformation theory of plasticity.

Figure 29 also portrays the results of the theoretical solution for n = 9. Comparison of the mathematical solution for n = 19 and n = 9 shows a large difference in strain concentration factors with increasing plastic flow. However, the variation of stress concentration factors with λ (as shown in the paper by Budiansky and Vidensek, for n = 19 and 9) shows a maximum difference of approximately 10%. The dependence of stress and strain concentration factors on the parameter n can be seen from the equations

$$K_\sigma = 3 - \frac{A_{00} + 2A_{01} + A_{10} + 2A_{11}}{\lambda}$$

$$K_\varepsilon = K_\sigma \left[\frac{1 + \frac{3}{7} (K_\sigma)^{n-1} (\lambda)^{n-1}}{1 + \frac{3}{7} (\lambda)^{n-1}} \right]$$

where: K_σ stress concentration factor $(\sigma_{max}/\sigma_{avg})$

K_ε strain concentration factor $(\varepsilon_{max}/\varepsilon_{avg})$

A_{ij} Fourier coefficients in the Budiansky-Vidensek solution (dependent on λ and n)

These equations show that K_ε is more sensitive to changes in n than is K_σ. Thus, the shape of the stress-strain curve has a great influence on the strain concentration factor, but less influence on the stress concentration factor. Ramberg-Osgood stress-strain curves of the Laminac polyester resin, as well as those of references 31-34, show that it is not possible to obtain an exact fit regardless of the value of the shape parameter n.

Therefore, it can be stated that, even though exact agreement between theoretical and experimental Ramberg-Osgood curves (for the materials considered) is not possible, one can obtain good agreement between theoretical and experimental stress concentration factors, but poor agreement for strain concentration factors.

Further comments concerning the theoretical solution will be made in the section concerning two-dimensional states of stress.

V. EXTENSION TO TWO-DIMENSIONAL PROBLEMS

All considerations so far have been made by assuming a uniaxial state of stress. Thus, it is desirable to consider the possibility of extending the proposed method to two-dimensional states of stress. A law is needed whereby two-dimensional states of stress and/or strain may be evaluated based on uniaxial calibration tests. However, before seeking such a law it is advantageous to perform exploratory tests to give insight into the direction of further experimentation.

The tests used to determine stress and strain concentration factors can also be used as preliminary testing for two-dimensional problems. As has been previously stated, it is desirable to have a material that exhibits large plastic strains. Concurrently, fringe density should not be so high that fringe orders can not be counted. This is a problem when using polycarbonate, as pointed out by Whitfield (26).

A plate was tested for which the maximum strain was approximately 30%. Figure 30 shows a dark field isochromatic photograph taken before complete fracture, while Fig. 31 portrays the same plate after fracture. These figures indicate that fringes are easily determined; enlargement of the photographs in the region of maximum fringe density yields a fringe pattern that is useable for analysis. Thus, the 60:40 mixture of Laminac polyester resins permits large strains together with fringe patterns that can be resolved.

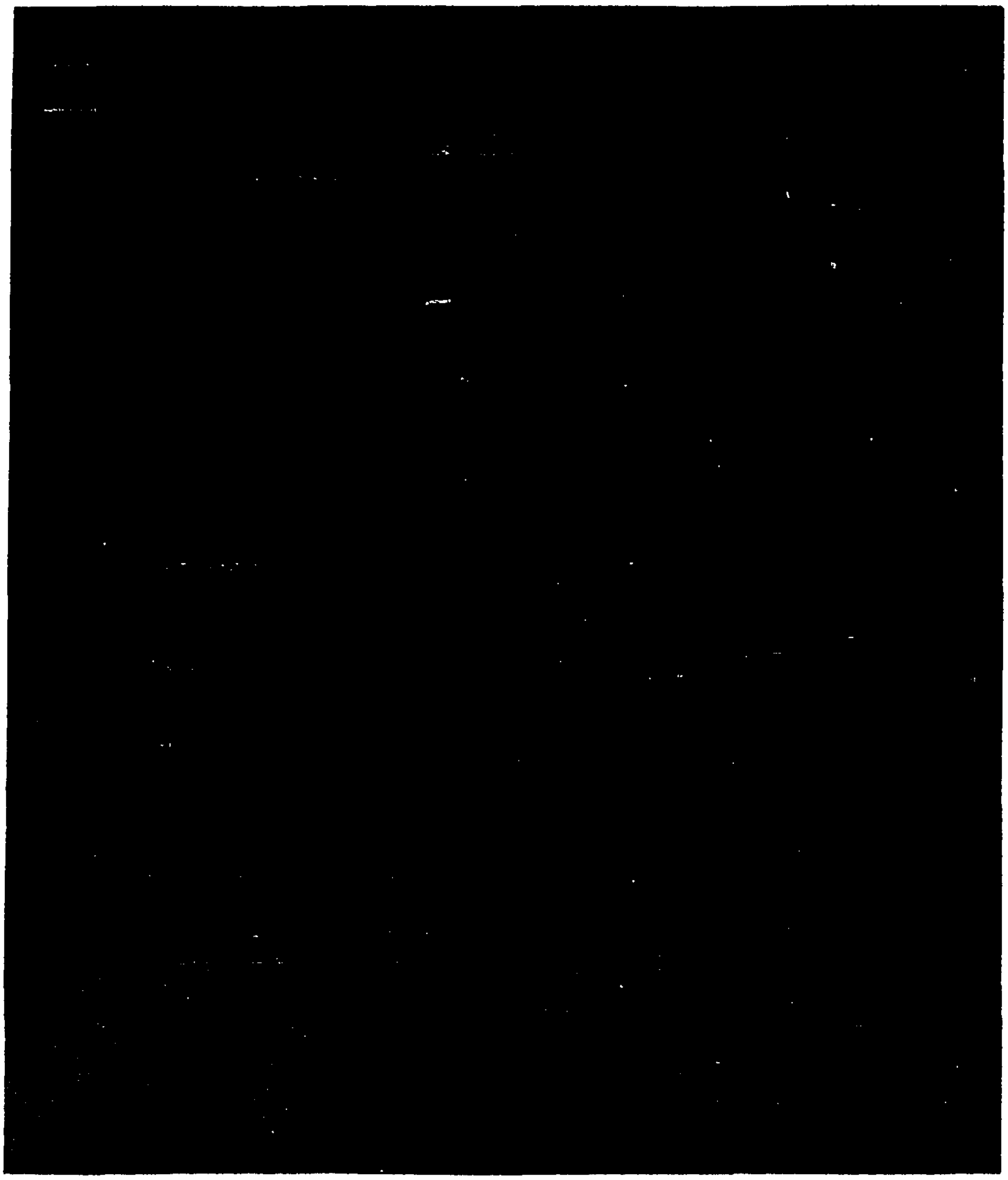

Fig. 30. Dark field isochromatic photograph before complete fracture

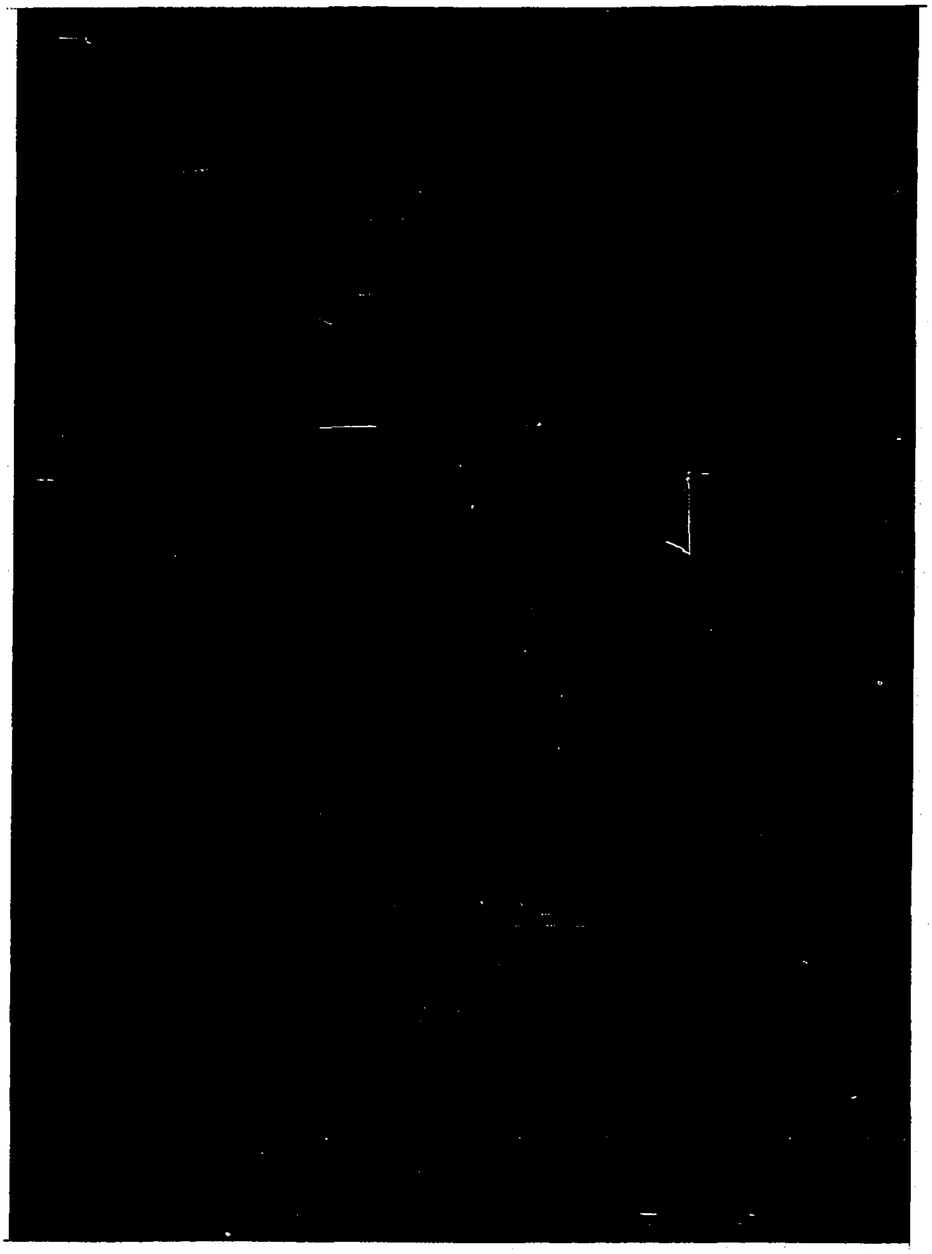

Fig. 31. Dark field isochromatic photograph after fracture

Careful inspection of Fig. 30 indicates that, in the cracked region, the birefringent pattern is not disturbed to any significant degree. Stress concentrations exist at the crack tip, whereas the remainder of the crack surface is a free surface. This tends to lend evidence that birefringence is a function of strain rather than stress.

Much basic research is necessary before proceeding to the evaluation of stresses and strains in two-dimensional states of stress. However, it is necessary to first ask the question: Is it worthwhile to continue this program, and do we have any indication of likely success? This indication is obtained by assuming that uniaxial calibrations are applicable to two-dimensional problems, and comparing results with those of other researchers. In particular, a comparison can be made between the proposed method and the work of Budiansky-Vidensek (31), and Durelli-Sciammarella (34).

As stated in the previous chapter, Budiansky and Vidensek gave a theoretical solution for stresses and strains in the plastic range around a circular hole in an infinite plate subjected to uniaxial tension. The solution was obtained by digital computer for the transverse axis through the hole, where both stresses and strains are principal. The theoretical equations were evaluated for $n = 19$, as shown in Fig. 18.

As previously stated, evidence tends to indicate that birefringence is strain dependent. However, the relationship between these two quantities is not known. An obvious relationship would be that of photoelasticity, that is, optical response is related to principal strain difference. Assuming this simple extension gives the results shown in Fig. 32. Principal strain differences for the polyester material were found using unload isochromatic photographs and uniaxial calibrations. In Fig. 32 a is the hole radius and r is a transverse distance measured from center of hole.

Experimental data and the theoretical solution do not exhibit good agreement. In the region away from the hole the theoretical solution gives increasing strain for increasing loads. However, in the plastic region near the hole strains increase up to some value of load and then decrease as the load is increased. As previously mentioned the theoretical solution is not valid where unloading occurs.

Good agreement is not displayed, even in the region of increasing strain with increasing load, except for $\lambda = 0.67$. Agreement, or lack of agreement between results is difficult to explain since the validity of the theoretical results is not positively known, and since uniaxial calibrations were assumed to extend to the present case.

A further check between experimental and theoretical results can be made by comparing the results of Durelli-Sciammarella (34) and Budiansky-Vidensek (31). This

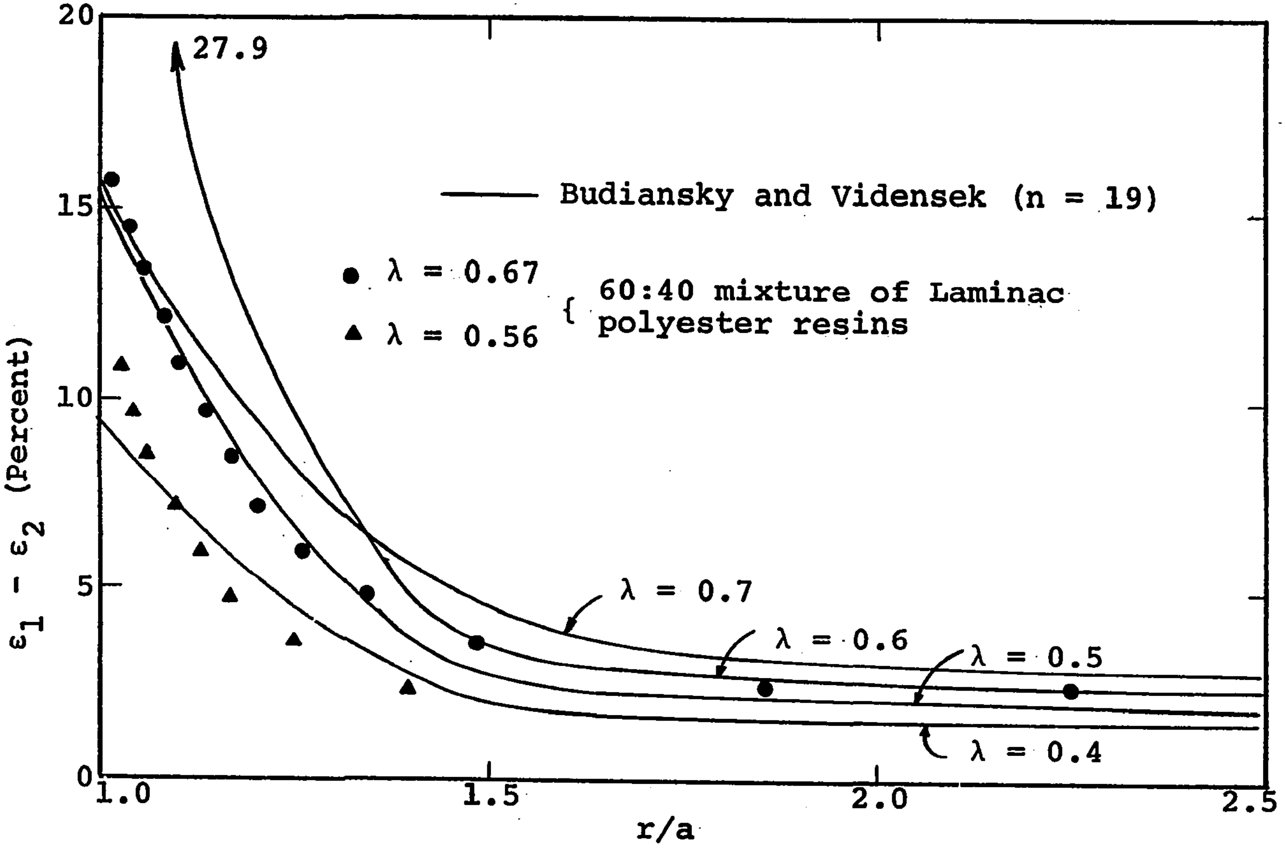

Fig. 32. Comparison of principal strain difference along transverse axis

comparison is shown in Fig. 33. The best Ramberg-Osgood fit is given by $n = 19$. Fair agreement exists for $\lambda = 0.4$, but the curves tend to deviate more for $\lambda = 0.5$. This may be due to the fact that, for increasing loads, the plate of Durelli and Sciammarella ceases to act as an infinite plate. However, inspection of the curves presented by Durelli shows that, for some applied load levels, the plate may be assumed to be of infinite width; that is, at a distance from the hole the stress approaches the average applied stress. Within this range it is possible to compare results. A value of $\lambda = 0.5$ appears to be the upper limit on the infinite plate assumption.

The shape parameter n also has an influence on the results. Using $n = 9$ gives a strain difference of 5.4 ($\lambda = 0.4$) and 6.9 ($\lambda = 0.5$) at $r/a = 1.0$. For $r/a \geq 1.5$ the strain differences are practically the same as those of $n = 19$. The theoretical results in the plastic region near the hole are thus highly dependent on the shape of the stress-strain diagram. A check on the stress results for $n = 19$ shows the same behavior as that of strain results, that is, a decrease in maximum stress after a certain load level has been reached.

Frocht and Thomson (14) also give a comparison between theoretical and experimental results. Their experiments give stresses instead of strains, and show good agreement between

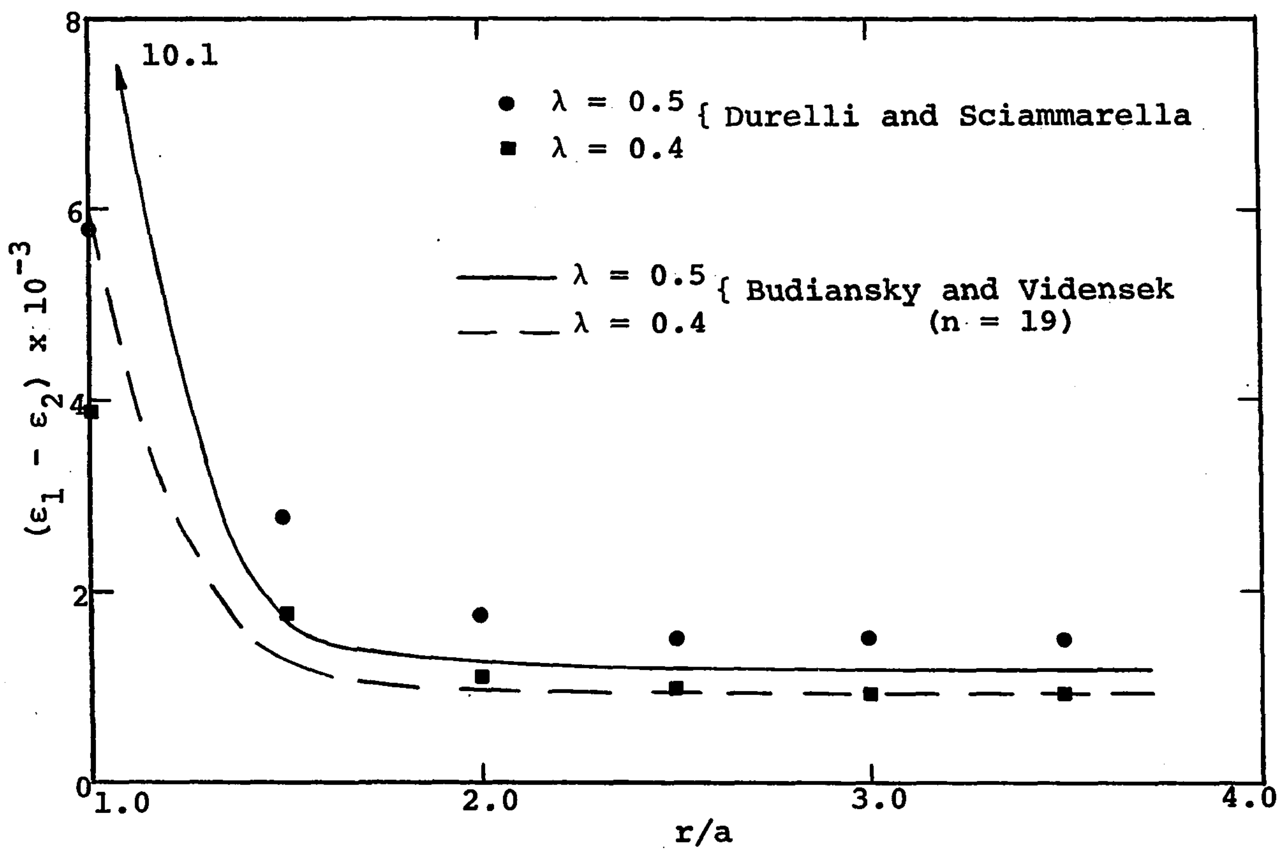

Fig. 33. Comparison of principal strain difference along transverse axis

theory and experiment except for the case of $\lambda = 0.9$, in the region of large plastic flow. They used a value of $n = 9$ for the Ramberg-Osgood stress-strain curve. They report,

> It will be noted that here ($\lambda = 0.9$) the theoretical σ_y curve (y being in the direction of the applied load) resembles closely the elastic distribution, i.e. the stress gradient is very large at the hole, whereas for $\lambda = 0.7$ the gradient is very small. Such a change in shape during plastic flow is quite unlikely. Budiansky and Vidensek point out that the numerical procedure they used is such that the accuracy of the final results cannot be positively assessed, and in view of the method used it seems likely that the results obtained by adding 'correction' stresses would be poorer in cases of large amounts of plastic flow than in cases of small amounts. This perhaps explains the discrepancy.

It is also interesting to note the conclusions of Budiansky and Vidensek,

> The solution is based on a stress-strain relation of questionable validity; and it is only approximate, even within the framework of the postulated theory.
>
> Of great interest would be another treatment of the problem via the simple flow, or incremental, theory of plasticity, and the subsequent comparison with the present results. Although the flow and deformation theories may differ substantially from one another for arbitrary stress paths, little is known about the effect of such deviations on the stress distributions that would be predicted by the two theories in problems such as the presently considered one.

In view of the results of all researchers reported in this investigation it is justifiable to say that the theoretical solution of Budiansky and Vidensek gives adequate stress concentration factors but poor strain concentration factors. This is true for all cases reported where an adequate Ramberg-Osgood stress-strain curve can be found. In some cases the theoretical and experimental stress distributions are in good agreement, whereas the distribution of strain is probably not adequate.

Further comparisons can be made with the work of Durelli and Sciammarella (34). They studied the problem of the determination of the distribution of stresses and strains in a plate with a circular hole under unidimensional load. An aluminum model was tested, with a plate width to hole diameter ratio of 6.4. This ratio for the photomechanics experiment was 13.3. Thus, it is apparent that there is a violation of the requirement of geometric similarity. This condition is necessary for a comparison of results. However, a previous discussion indicates that it is possible to compare results for some load levels.

According to Brill (23) a comparison can be made between results for the differences of principal strains along the transverse axis of symmetry of the specimen. Non-dimensional strain differences for aluminum and polyester are assumed to be the same,

$$[\frac{E}{\sigma_1}(\varepsilon_1-\varepsilon_2)]_P = [\frac{E}{\sigma_1}(\varepsilon_1-\varepsilon_2)]_A \qquad (2)$$

where P denotes polyester and A aluminum. This equation gives a means for predicting $(\varepsilon_1 - \varepsilon_2)_A$. As before, principal strain differences for the polyester were found from isochromatic photographs and uniaxial calibrations. Results are compared in Fig. 34. Good agreement exists at the edge of the hole where the state of stress is uniaxial, whereas fair agreement is shown away from the hole where the maximum normal stress approaches the average applied stress. However, very poor agreement exists elsewhere. This large deviation may be the result of several factors.

First, the uniaxial calibrations were assumed to extend to two-dimensional states of stress; further calibrations are necessary to validate this assumption. Second, geometric similarity was violated; further testing should be conducted on plates that are geometrically similar. Third, equation (2) was assumed to be applicable to the present problem. Further experimentation is needed to validate Brill's method for the polyester mixture. And last, errors arise due to model distortion. Prototype distortions are less than model distortion, resulting in errors due to changes in shape of the model.

Nonetheless, further studies of Laminac polyester resins are warranted. Possible areas of future research will be discussed in a later chapter.

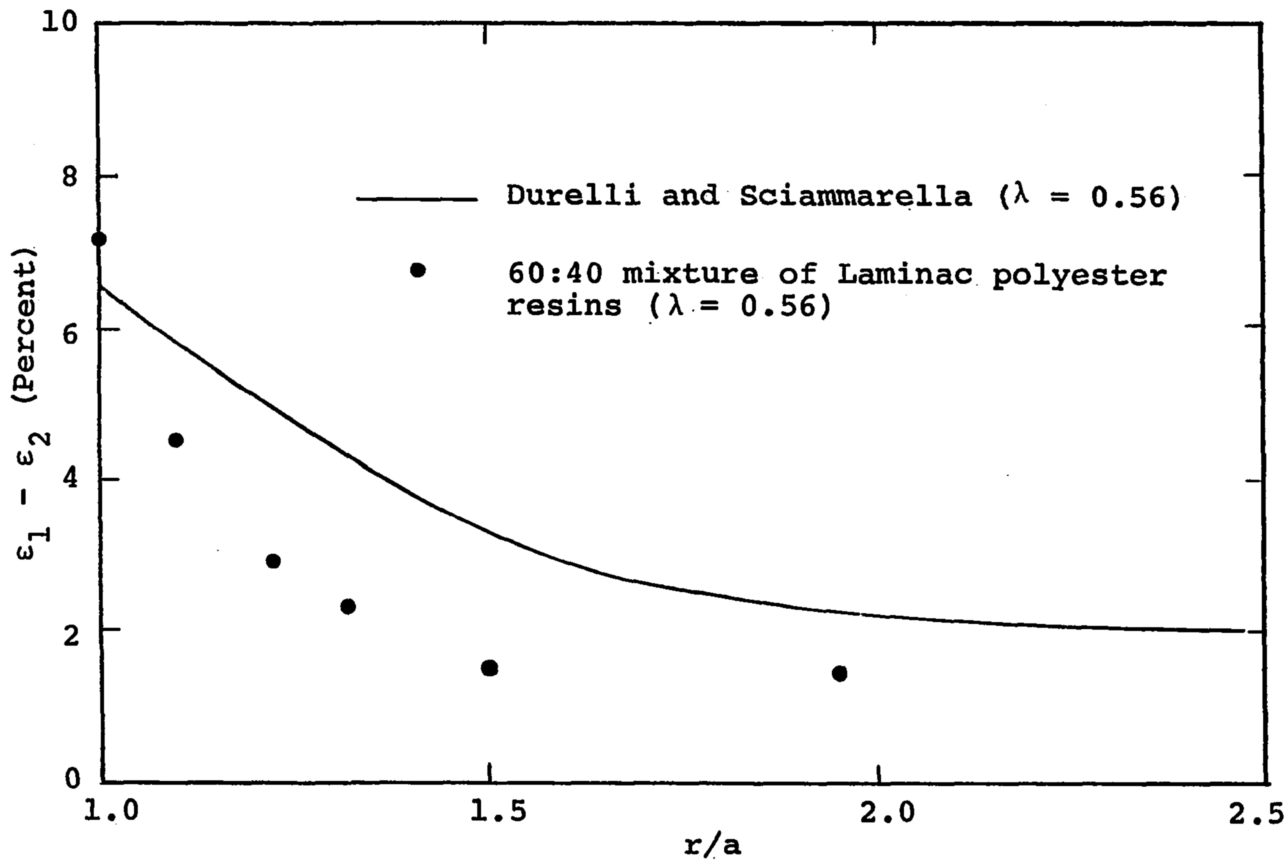

Fig. 34. Comparison of principal strain difference along transverse axis

VI. CONCLUSIONS AND RECOMMENDATIONS

A primary objective of this investigation was to find a material that could be used to determine stresses and/or strains in elasto-plastic problems. In particular, it was desirable that the material be able to undergo large plastic strains while at the same time exhibiting fringes that could be counted.

A 60:40 mixture of Laminac polyester resins gave the desired results. Uniaxial calibrations indicated the existence of a minimal strain rate such that curves of birefringence-stress-strain developed at this rate or less are virtually the same. Time effects were therefore eliminated, allowing the material to be used for the study of elasto-plastic problems.

The present investigation shows that it is possible to determine stresses and strains in one-dimensional stress fields, and in particular, it provides a means for evaluating stress and strain concentration factors.

A complete study of biaxial or triaxial states of stress is not yet possible. Further material calibration is necessary to study this problem.

Application of the proposed technique means that similitude requirements must be met. These conditions, as stated by Frocht (14), are not strictly adhered to. The materials (polyester and aluminum) do not have exactly the same Poisson's ratio, nor do they have the same shape of stress-strain diagram. The condition of same yield criterion was not investigated. The requirement of similarity of geometry was also violated in some cases, but all cases studied had plates that approached the infinitely wide plate.

Nonetheless, the 60:40 mixture of polyester resins was found to be applicable to the solution of stress and strain concentration factor problems. The usefulness of the material and method to two-dimensional problems awaits further study.

It should be pointed out that stresses and strains for the polyester were based upon original dimensions. In addition, the influence of reduction in thickness on birefringence per unit thickness was neglected. This leads to errors in maximum strain of the order of 10% and maximum stresses of the same order.

Considering all sources of error, stress and strain concentration factor comparisons show that the proposed method is reliable and gives results that are similar to those of other methods.

Several suggestions can be made concerning future research. Calibrations in two-dimensional stress fields should be the first consideration. From this one could determine whether birefringence is a function of stress, strain, or both.

One might also wish to compare experimental results with a recent theory of Morris (35). His theory shows that the difference in indices of refraction is not simply a function of difference of principal strains (as in photoelasticity), but is a function of difference and sum of principal strains. Results of the present investigation could not be compared to the theory due to lack of knowledge of several material constants.

There is a need for investigation of a one-dimensional state of compressive stress. A study into the meaning of isoclinic parameters would also be of interest. A fruitful area of investigation would be the determination of a yield criterion.

In summary, it can be stated that this investigation achieved the following:

1. A material that permits large strains and suitable optical response was found.
2. A technique was developed whereby optical response can be used to measure strains.
3. The proposed technique was used to compare photomechanics results (stress and strain concentration factors) with those of other methods. The results compared favorably with data from strain gage, moiré, and optical grid measurements.

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